RK INSTITUTEOF ENGINEERING AND TECHNOLOGY

(Department of Civil Engineering)

LECTURE NOTE

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Degree : DIPLOMA

Semester : 3RD

Name of Subject : STRUCTURAL MECHANICS

Subject Credit : 3

CHAPTER-2

SIMPLE STRESS AND STRAIN:

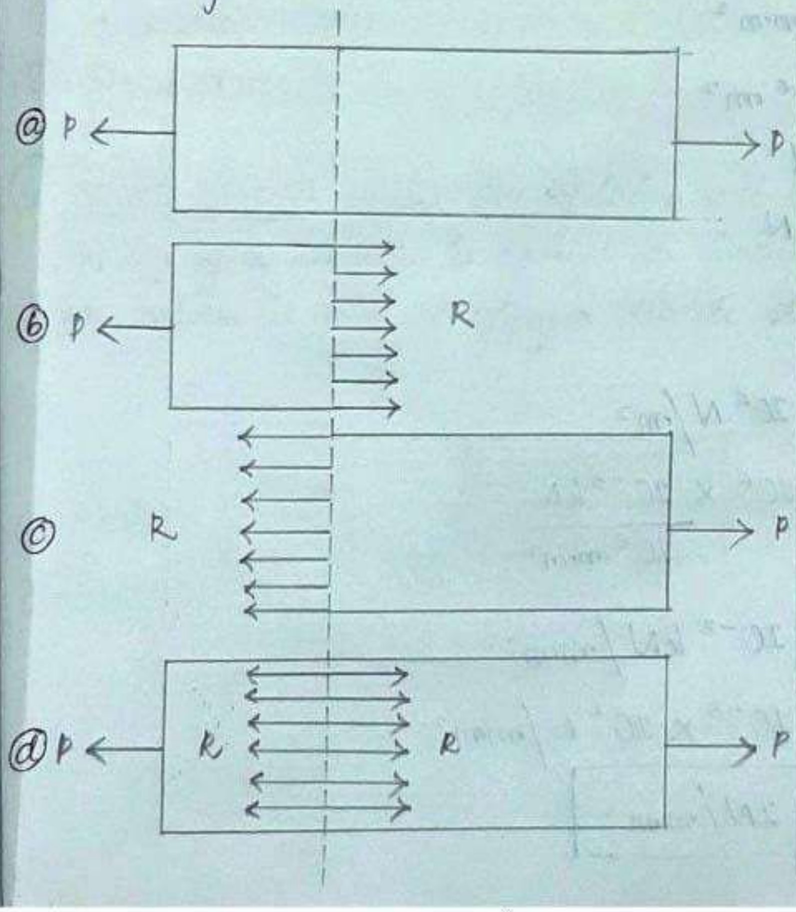
* Stress: A resistance force offered by a body against the deformation is called stress.
(change in shape and size)

② The external force acting on the body is called load.

③ In a single line, load is applied on the body and stress is induced in the material of the body.

④ Stress is denoted by 'σ' (sigma).

⑤ Let a rod of uniform cross-sectional area is subjected to pulling force (P), to resist the deformation, then an internal force (R), will be induced inside the material.



$\therefore$ Stress $= \frac{R}{A}$

$$\boxed{\sigma = \frac{P}{A}} \quad [\because R = P]$$

→ Unit : $\frac{\text{Newton}}{(\text{metre})^2}$

$= \frac{N}{m^2}$ = Pascal

$\boxed{R=P}$ no tearing of the body

[applied force = resisting force]

Stress $= \frac{\text{Internal resisting force}}{\text{area}}$

UNITS OF STRESS

Kilo = 10^3
Mega = 10^6
Giga = 10^9
Tera = 10^{12}
Milli = 10^{-3}
Micro = 10^{-6}
Nano = 10^{-9}
Pico = 10^{-12}

1 Pascal = $1\ N/m^2$.

1 KiloPascal = $10^3\ N/m^2$.

1 Mega Pascal = $10^6\ N/m^2$.

1 Giga Pascal = $10^9\ N/m^2$

* $1\,m \rightarrow 10^3\,mm$

  $1\,mm \rightarrow \frac{1}{10^3}\,m = 10^{-3}\,m$

* $1\,m^2 = (10^3)^2\,mm^2$

  $= 10^6\,mm^2$.

* $1\,mm^2 = 10^{-6}\,m^2$.

* $1\,kN = 10^3\,N$

* $1\,N = 10^{-3}\,kN$

We know,

$$1\,MPa = 10^6\,N/m^2$$

$$= 10^6 \times \frac{10^{-3}\,kN}{10^6\,mm^2}$$

$$= 10^{-3}\,kN/mm^2.$$

$$= 10^{-3} \times 10^3\,N/mm^2.$$

$$\boxed{1\,MPa = 1\,N/mm^2.}$$

* **Strain:** This is the ratio of change in dimension to the original dimension.

→ It is denoted by 'e'.

$$\Rightarrow \boxed{e = \frac{\Delta l}{l}} \Rightarrow \frac{\text{change in length}}{\text{original length}} \Rightarrow \frac{\Delta d}{d} \Rightarrow \frac{\Delta v}{v}$$

linear strain [→ tensile, → compressive] | lateral strain | volumetric strain

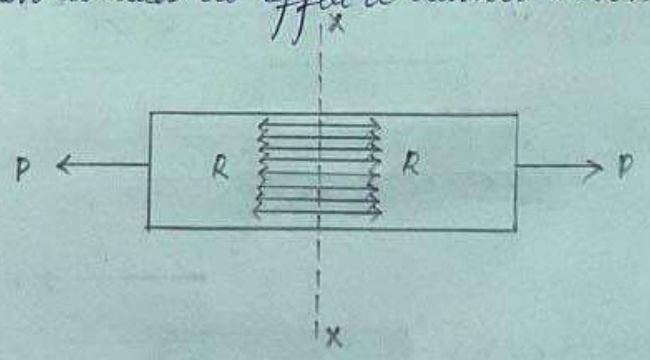
→ Strain has no units i.e. it is unitless.

* **Types of Stress:**

A material is capable of offering three types of stresses:

(i) Tensile stress
(ii) Compressive stress
(iii) Shear stress

(i) **Tensile Stress:** When the offering resistance by a section of a member is against an increase in length, the section is said to offer a tensile stress.



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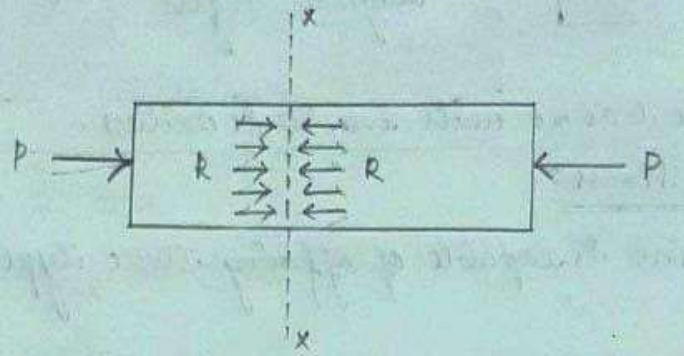
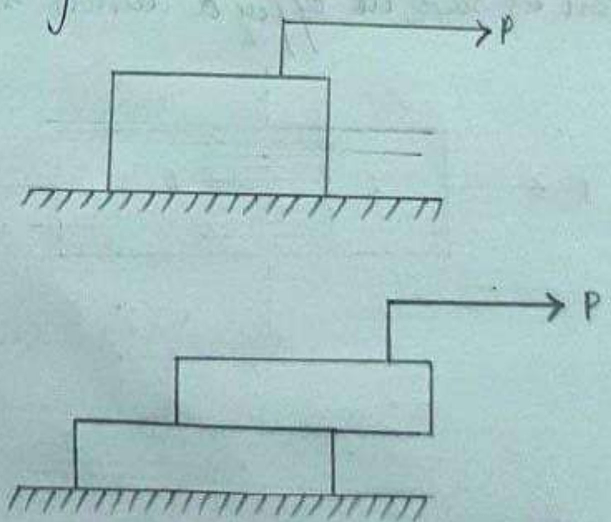
(ii) Compressive Stress: When the offering resistance by a section of a member is against a decrease in length, the section is said to offer a compressive stress.



(iii) Shear Stress: (τ) A load P is applied tangentially along the top of the face of the body of which, bottom face is fixed. Such force acting tangentially along a surface is called as shear force.



→ The resistance provided in this case is called shear resistance.

→ Shear stress is defined as the ratio of shear resistance to the shear area.

mathematically, $\boxed{\tau = \frac{R}{A}}$

*** Types of strain:** 01.11.2021

Strain can be of 4 types:

(i) Tensile strain: The ratio of increase in length to its original length, is called tensile strain.

$\boxed{e = \frac{\Delta l}{l}}$ → here, Δl → change in increase in length

(ii) Compressive strain: The ratio of decrease in length to the original length, is called compressive strain.

$\boxed{e = \frac{\Delta l}{l}}$ → here, Δl → change in decrease in ~~leg~~ length

(iii) Lateral strain: The ratio of the change in lateral dimension to the original lateral dimension, is called lateral strain.

$\boxed{e = \frac{\Delta d}{d}}$ → here, Δd → change in lateral dimension

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(iv) <u>Volumetric strain</u>: The ratio of the change in volume to its original volume, is called as volumetric strain.

$$\boxed{e = \frac{\Delta v}{v}}$$ → here, Δv → change in volume

* <u>Hooke's Law</u>: (most imp) → 2 marks regular question

It states that "when a material is loaded such that the intensity of stress within a certain limit, the ratio of the intensity of the stress to the corresponding strain is a constant."

$$\frac{\text{stress}}{\text{strain}} = \frac{\sigma}{e} = \text{constant}.$$

$$\Rightarrow \frac{\sigma}{e} = \text{constant}$$

$$\Rightarrow \sigma = \text{constant} \times e$$

$$\Rightarrow \boxed{\sigma \propto e}$$

→ It also can be stated as "Stress is directly proportional to strain, within a certain limit."

* Modulus of Elasticity (or) Young's Modulus:

In case of axial loading, the ratio of the intensity of tensile or compressive stress to the corresponding strain is constant.

→ This ratio is called modulus of elasticity or young's modulus and is denoted by 'E'.

$$\frac{\sigma}{e} = \text{constant} = E$$

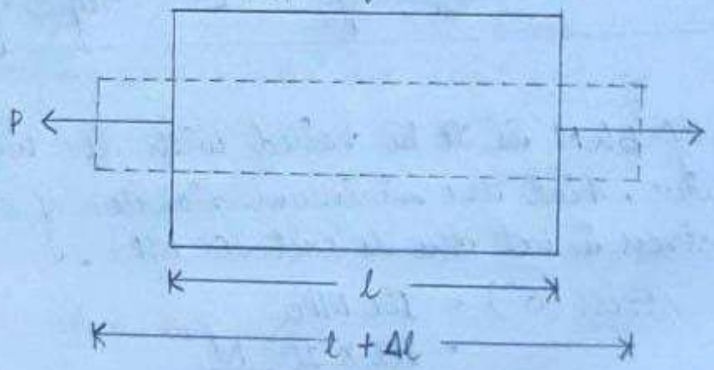
$$\Rightarrow \boxed{E = \frac{\sigma}{e}}$$

[∵ constant is nothing but the young's modulus 'E'.]

* Deformation of prismatic bar due to uni-axial load:

prismatic → throughout the length no change in dimension or no change in cross-sectional area.

uni-axial → 1 or same.



Consider a prismatic bar is subjected to axial tensile load 'P' → same as an external force.

The stress will be introduced due to load P is given by, $\boxed{\sigma = \frac{P}{A}}$

we know, strain : $e = \frac{\Delta l}{l}$

According to Hooke's law,

$$\sigma \propto e$$

$$\Rightarrow \sigma = Ee$$

$$\Rightarrow \frac{P}{A} = E\frac{\Delta l}{l} \qquad \left[\because \sigma = \frac{P}{A} \text{ and } e = \frac{\Delta l}{l}\right]$$

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$$\Rightarrow \boxed{\Delta l = \frac{Pl}{AE}}$$

→ In the above formula, the term 'AE' is known as axial rigidity. [AE → axial rigidity]

Questions:

① A load of 5kN is to be raised with the help of a steel wire. Find the minimum diameter of a wire, if the stress is not ~~the~~ to exit 100 MPa.

→ given: stress (σ) = 100 MPa
$= 100 \times 10^6 \frac{N}{m^2}$.

external force (P) = 5kN
$= 5 \times 10^3 N$

$$\sigma = \frac{P}{A}$$

$$\Rightarrow 100 \times 10^6 = \frac{5 \times 10^3}{\frac{\pi}{4} d^2}$$

$$\Rightarrow d^2 = \frac{5 \times 10^3}{100 \times 10^6 \times \frac{\pi}{4}}$$

$$\Rightarrow d = \sqrt{\frac{5 \times 10^3}{100 \times 10^6 \times \frac{\pi}{4}}}$$

$$\Rightarrow \boxed{d = 7.97 \times 10^{-3} m}$$

$$\Rightarrow d = 7.97 \times 10^{-3} \times 10^3 \text{ mm}$$

$$\Rightarrow \boxed{d = 7.97 \text{ mm}}$$

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② A steel rod 500 mm long and 20 mm × 10 mm in cross-section is subjected to axial pull of 300 kN. If modulus of elasticity is $2 \times 10^5 \frac{N}{mm^2}$. Calculate the elongation of the rod.
Also calculate the strain induced in the bar.

→ given: $l = 500$ mm

$P = 300 kN = 300 \times 10^3 N$

$E = 2 \times 10^5 N/mm^2$

$A = 20 mm \times 10 mm$
$= 200 mm^2$

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$\therefore \underline{\Delta l}$

$$\therefore \Delta l = \frac{Pl}{AE} = \frac{300 \times 10^3 \times 500}{200 \times 2 \times 10^5} = 3.75 \text{ mm}$$

$$\text{and strain} = \frac{\Delta l}{l} = \frac{3.75}{500} = 7.5 \times 10^{-3}.$$

② A hollow cylinder 2m long has an outside diameter of 50 mm and inside diameter of 30 mm. If the cylinder is carrying a load of 25 kN, find the stress in cylinder, also find deformation of the cylinder. Take $E = 100\,GPa$.

→ given: $l = 2m = 2 \times 1000 \text{ mm} = 2000 \text{ mm}$

$D = 50 \text{ mm}$

$d = 30 \text{ mm}$

$E = 100\,GPa$

$P = 25\,kN = 25 \times 10^3\,N$

$\therefore$ Area of hollow cylinder : $A = \frac{\pi}{4}(D^2 - d^2)$

$\Rightarrow A = \frac{\pi}{4}\left((50)^2 - (30)^2\right)$

$\Rightarrow \boxed{A = 1256 \text{ mm}^2}$

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$1m \rightarrow 10^3 mm$
$1m^2 \rightarrow (10^3)^2 mm^2$
$\boxed{1m^2 = 10^6 mm^2}$

$$E = 100 \, GPa$$
$$= 100 \times 10^9 \frac{N}{m^2}$$
$$= 100 \times 10^9 \frac{N}{10^6}$$
$$= 100 \times 10^9 \times 10^6 \frac{N}{mm^2}$$
$$E = 100 \times 10^3 \frac{N}{mm^2}$$

$$\therefore \sigma = \frac{P}{A} = \frac{25 \times 10^3}{1256} = 19.90 \frac{N}{mm^2}$$

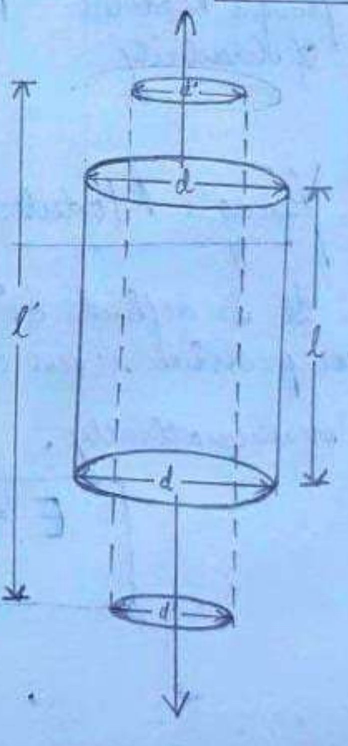
$$\text{and } \Delta l = \frac{Pl}{AE} = \frac{25 \times 10^3 \times 2000}{1256 \times 100 \times 10^3} = 0.398 \, mm.$$

* POISSON'S RATIO $\left(\mu / \frac{1}{m}\right)$

05.11.2021

linear strain $= + \frac{l'}{l}$

lateral strain $= - \frac{d'}{d}$



→ Poisson's ratio is defined as "the ratio between the lateral strain to the longitudinal strain."

→ It is denoted as μ or $\frac{1}{m}$ and the value of μ is always less than 1.

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mathematically,

$$\mu = \frac{-\text{lateral strain}}{\text{longitudinal strain}}$$

→ It is a constant and it has no unit.

→ ex: (i) Poisson ratio of concrete = 0.15 to 0.3
(ii) Poisson ratio of steel = 0.27 to 0.3

→ Poisson ratio varies from $\boxed{-1 \text{ to } 0.5}$

Elastic Constant

E: Young's Modulus of Elasticity

K: Bulk Modulus

C: Shear Modulus (or) Modulus of Rigidity

(i) Young's Modulus (E):

It is defined as "the ratio of tensile or compressive stress to the corresponding strain."

mathematically,

$$\boxed{E = \frac{\sigma}{e}}$$

(ii) Bulk Modulus (K) :

when an uniform element is subjected to equal stress in 3 mutually perpendicular directions then "the ratio of direct stress to the volumetric strain is known as bulk modulus."

$\left[\because e_v = \frac{\Delta V}{V}\right]$ volumetric strain

mathematically,

$$\boxed{K = \frac{\sigma}{e_v}}$$

(iii) Shear Modulus (C or G) :

It is defined as "the ratio of shear stress to the shear strain."

→ Higher the value of shear modulus, shows that the body is highly rigid.

mathematically,

$$\boxed{C = \frac{\tau}{\epsilon}}$$

where, τ → shear stress
ϵ → shear strain

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* Relationship between Elastic Constants E, K, C:

(i) Relationship between E and K:

$$\boxed{E = 3K\left(1 - \frac{2}{M}\right)}$$

(ii) Relationship between E and C:

$$\boxed{E = 2C\left(1 + \frac{1}{m}\right)}$$

where, $\frac{1}{m}$ is poisson's ratio

(iii) Relationship between E, K and C:

$$\boxed{E = \frac{9KC}{3K + C}}$$

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* **Proof:** $\boxed{E = \dfrac{9KC}{3K+C}}$

we know,

$$E = 3K\left(1-\frac{2}{M}\right) \Rightarrow \boxed{1-\frac{2}{M} = \frac{E}{3K}} \text{ ——— (1)}$$

$$E = 2C\left(1+\frac{1}{m}\right) \Rightarrow \boxed{1+\frac{1}{m} = \frac{E}{2C}} \text{ ——— (2)}$$

multiplying 2 in eq^n (2)

$$2\left(1+\frac{1}{M}\right) = \cancel{2}\left(\frac{E}{\cancel{2}C}\right)$$

$$\Rightarrow \boxed{2+\frac{2}{M} = \frac{E}{C}} \text{ ——— (3)}$$

now adding (1) and (3)

$$1-\cancel{\frac{2}{M}}+2+\cancel{\frac{2}{M}} = \frac{E}{3K}+\frac{E}{C}$$

$$\Rightarrow 1+2 = \frac{E}{3K}+\frac{E}{C}$$

$$\Rightarrow 3 = \frac{EC+E3K}{3KC}$$

$$\Rightarrow 3(3KC) = EC+E3K$$

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$\Rightarrow 9KC = E(C+3K)$

$\Rightarrow \boxed{E = \dfrac{9KC}{3K+C}}$ hence proved.

Questions:

① The modulus of rigidity of a material is $0.8 \times 10^5 \frac{N}{mm^2}$. find the poisson's ratio if the modulus of elasticity of that material is $2.1 \times 10^5 \frac{N}{mm^2}$.

→ given: $C = 0.8 \times 10^5 \frac{N}{mm^2}$

$E = 2.1 \times 10^5 \frac{N}{mm^2}$

μ or $\frac{1}{M} = ?$

$\therefore E = 2C\left(1+\frac{1}{M}\right)$

$\Rightarrow 2.1 \times 10^5 \frac{N}{mm^2} = 2 \times 0.8 \times 10^5 \frac{N}{mm^2}\left(1+\frac{1}{M}\right)$

$\Rightarrow \dfrac{2.1 \times 10^5}{2 \times 0.8 \times 10^5} = 1 + \dfrac{1}{M}$

$\Rightarrow \dfrac{2.1 \times 10^5}{2 \times 0.8 \times 10^5} - 1 = \dfrac{1}{M}$

$\Rightarrow \dfrac{1}{M}$ or $\mu = \dfrac{2.1 \times 10^5}{2 \times 0.8 \times 10^5} - 1.$

$\Rightarrow \boxed{\mu = 0.3125}$ - Ans

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(2) The modulus of rigidity of material is $0.8 \times 10^5 \ \frac{N}{mm^2}$ when a 6mm × 6mm rod of this material was subjected to an axial pull of 3600 N. It was found that the lateral dimension of the rod changed to 5.9991 mm × 5.9991 mm. Find the poisson's ratio and modulus of elasticity of that rod.

→ given: $C = 0.8 \times 10^5 \ \frac{N}{mm^2}$.

$A = 6\,mm \times 6\,mm$
$= 36\,mm^2$

$P = 3600\,N$

$$\sigma = \frac{P}{A} = \frac{3600}{36} = 100 \ \frac{N}{mm^2}$$

longitudinal strain, $E = \frac{\sigma}{e} \rightarrow \boxed{e = \frac{\sigma}{E}}$

$$\text{lateral strain} = \frac{\text{change in lateral dimension}}{\text{original dimension}}$$

$$= \frac{6 - 5.9991}{6} = 0.00015$$

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Poisson's ratio:

$$\frac{1}{M} = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\Rightarrow \frac{1}{M} = \frac{0.00015}{\frac{\sigma}{E}}$$

$$\Rightarrow \frac{1}{M} = \frac{0.00015}{100} \times E$$

$$\Rightarrow \frac{1}{EM} = \frac{0.00015}{100}$$

$$\Rightarrow ME = \frac{100}{0.00015}$$

$$\Rightarrow \boxed{ME = \frac{2\times 10^6}{3}} \quad (1)$$

we know, $E = 2C\left(1+\frac{1}{M}\right)$

$$\Rightarrow \cancel{M}E = 2C \left(\frac{M+1}{M}\right)$$

$$\Rightarrow ME = 2C (M+1)$$

$$\Rightarrow \boxed{ME = 2\times 0.8\times 10^5 (M+1)} \quad (2)$$

equating eq^n (1) and (2)

$$\Rightarrow 2\times 0.8\times 10^5 (M+1) = \frac{2\times 10^6}{3}$$

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$$\Rightarrow M + 1 = \frac{2 \times 10^6}{3(2 \times 0.8 \times 10^5)}$$

$$\Rightarrow M = \frac{2 \times 10^6}{3(2 \times 0.8 \times 10^5)} - 1$$

$$\Rightarrow \boxed{M = 3.167}$$

$$\because \mu = \frac{1}{M} = \frac{1}{3.167}$$

$$\Rightarrow \boxed{\mu = 0.315}$$

$$\therefore E = 2C\left(1 + \frac{1}{M}\right)$$

$$\Rightarrow E = 2 \times 0.8 \times 10^5 (1 + 0.315)$$

$$\Rightarrow E = 210400$$

$$\Rightarrow \boxed{E = 2.1 \times 10^5 \frac{N}{mm^2}}$$

08.11.21

* Axial deformation of bar due to its self-weight:

Considering a prismatic bar of uniform cross-sectional of length 'l' suspended freely.

weight of the bar = W

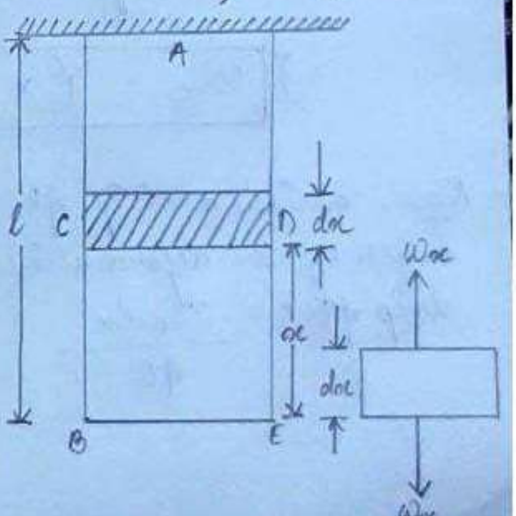
unit weight of the bar = Y

$$\left[\begin{array}{l} Y = \dfrac{W}{\text{volume}} \\ \Rightarrow Y = \dfrac{W}{Ax} \end{array}\right]$$

$A \rightarrow$ area

$x \rightarrow$ length

$Ax \rightarrow$ volume



Assuming cross-sectional area = 'A'
Consider a small element of length 'dx' at a distance 'x' from the free end.
The element is subjected to weight of bar below element (i.e. BCDE)

weight of bar below element dx

$$\Rightarrow W_x = Y(Ax)$$

Elongation of elemental length dx

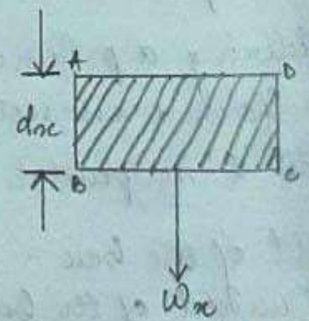
$$\Rightarrow \boxed{d\Delta = \frac{W_x\,dx}{AE}}$$

Let E, ρ, Y, be the Young's Modulus, density and unit weight of the body respectively.

we know, (volume) $V_x = A \times x$

$$Y = \frac{W_x}{V_x}$$

$$\Rightarrow \boxed{W_x = Y \times V_x}$$



Now, consider a strip ABCD of length by the deformation for the strip ABCD = $\frac{W_x\,dx}{AE}$

Total deformation of the bar = $\int_0^l \frac{\omega_x \times dx}{AE}$

$$= \int_0^l \frac{Y \times V_x}{AE} dx$$

$$= \int_0^l \frac{Y \times \cancel{A} \times x}{\cancel{A} E} dx$$

$$= \int_0^l \frac{Y x}{E} dx$$

$$= \frac{Y}{E} \int_0^l x \, dx \quad \left[\because Y \text{ and } E \text{ are constants}\right]$$

$$= \frac{Y}{E} \left(\frac{x^2}{2}\right)_0^l$$

$$= \frac{Y}{E} \left[\frac{l^2}{2} - \frac{0^2}{2}\right]$$

$$= \frac{Y}{E} \times \frac{l^2}{2}.$$

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$$\Rightarrow \boxed{\Delta = \frac{\gamma l^2}{2E}}$$

$\Delta l \rightarrow$ total deformation of the bar.

* **NOTES**:

① Ultimate stress $= \dfrac{\text{maximum load}}{\text{Area}}$

② Yield stress $= \dfrac{\text{yield point load}}{\text{Area}}$

③ Safe stress $= \dfrac{\text{yield stress}}{\text{load factor}}$

Questions:

① A following data refers to a mild steel specimen tested in a laboratory:

(i) diameter of specimen = 25 mm
(ii) length of specimen = 300 mm
(iii) extension under a load 50 kN = 0.045 mm
(iv) load at yield point = 127.65 kN
(v) maximum load = 208.60 kN
× (vi) length of the specimen after failure = 375 mm
× (vii) neck diameter = 17.75 mm

Now, determine: (i) Young's Modulus
(ii) Yield point stress
(iii) Ultimate stress
(iv) Safe stress adopting a factor of safety of 2.

→ given: $d = 25\ mm$

$l = 300\ mm$

$P = 50\ kN$

$= 50 \times 10^3\ N$

$\Delta l = 0.045\ mm$

yield point load $= 127.65\ kN$

$= 127.65 \times 10^3\ N$

maximum load $= 208.60\ kN$

$= 208.60 \times 10^3\ N$

load factor $= 2$.

(i) $\Delta l = \dfrac{Pl}{AE}$

$\Rightarrow E = \dfrac{Pl}{A \Delta l}$

$\Rightarrow E =$

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(ii) Yield point stress $= \dfrac{\text{yield point load}}{\text{area}}$

$$= \frac{127.65 \times 10^3 \ N}{\frac{\pi}{4}(25)^2 \ mm^2}$$

$$= 260.05 \ \frac{N}{mm^2}$$

(iii) Ultimate stress $= \dfrac{\text{maximum load}}{\text{area}}$

$$= \frac{208.60 \times 10^3 \ N}{\frac{\pi}{4}(25)^2 \ mm^2}$$

$$= 424.96 \ \frac{N}{mm^2}.$$

(iv) Safe stress $= \dfrac{\text{yield stress}}{\text{load factor}}$

$$= \frac{260.05}{2} \ \frac{N}{mm^2}.$$

$$= 130.025 \ \frac{N}{mm^2}.$$

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Mechanical Properties Of Materials

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MECHANICAL PROPERTIES OF MATERIALS :

① Rigidity:

(i) This is defined as the property possessed by a solid body to change its shape.

(ii) It means when an external force is applied to the solid material, there won't be any change in its shape due to intermolecular attraction by the closely packed particles.

(iii) This is the property of the material to resistance the bending.

② Elasticity:

(i) This is the property of a body by virtue of which in return its original shape after removal of external force causing deformation which is applied on it.

(ii) Elastic property entirely depends upon the type of material and not on the shape and size.

③ Plasticity:

(i) This is the ability of a solid material to undergo permanent deformation when force is applied to it.

(ii) The ability of a material to retain the changed shape under application of load is known as plasticity.

(iii) Plastic deformation is the property of ductile and malleable solids.

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(4) Compressibility:

(i) This is the property of material by virtue of which it tends to flatten and reduce in size under pressure.

(ii) This nature or property of material changes the inter-molecular structure of the material.

(5) Hardness:

(i) The property of material by virtue of which it resists the local surface deformation when undergoes abrasion, drilling, impact, etc.

(ii) It is the state of material, being hard for which it can withstand friction.

(6) Toughness:

(i) The amount of energy per unit volume that a material can absorb before rupture is called toughness.

(ii) It can be defined as the ability of a material to resist breaking when force is applied to it.

(iii) This property allows the material to deform before rupture or fracture.

(7) Stiffness:

(i) The property of material which resists deformation when a force is applied to it. This is the rigidity of a material.

(ii) The material having more flexibility has less stiffness.

(iii) A stiff material has high young modulus.

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8) Brittleness:

(i) The property of a material by which it fractures when subjected to stress without deformation.

(ii) Brittle material has a little tendency to deform before rupture.

(iii) It has small plastic region.

Ex: Bone, concrete, ceramic, cast iron, glass products, etc.

9) Ductility:

(i) It is defined as the ability of a material to undergo permanent deformation through elongation and reduction in cross-sectional area or bending at room temperature without fracturing.

(ii) This ~~or~~ is an ability to undergo vast permanent deformation in tension.

Example of ductile material → copper, aluminium, steel.

★ $\boxed{\text{Opposite to ductile is brittle.}}$

10) Malleability:

(i) This is the property of material by which it can be beaten to form its thin sheets.

Ex: lead, tin, gold, silver, aluminium, copper, iron.

11) Creep:

(i) This is the permanent change in shape and size of a material which increases as a function of time under application of load and elevated temperature.

(ii) Creep is time independent.

(iii) Creep begins at different temperature for different material.

(12) Fatigue:

(i) This is the deterioration of the material subjected to repeated cycle of stress and strain resulting in progressive cracking, eventually producing fracture.

(ii) This ~~the~~ is the weakening of a material caused by cyclic loading that results in progressive damage and the growth of cracks.

(iii) This is responsible for 90% of mechanical failure.

(13) Durability:

(i) The ability of a material to remain serviceable during the useful time without damaging the material.

(ii) It represents how long the material works.

(14) Tenacity:

→ The property of material to resist the breaking ~~it~~ is known as tenacity.

★ **NOTES**:

(1) Percentage of Elongation:

(a) It is a measure of ductility.

(b) This can be obtained as $= \boxed{\frac{\text{final length} - \text{initial length}}{\text{initial length}} \times 100}$

or $\boxed{\frac{\Delta l}{l} \times 100}$

where, Δl = change in length

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(2) Percentage of reduction in area:

(a) This is the measure of the specimen that how much that specimen narrowed when it undergoes some load application.

(b) It is obtained as follows:

$$\boxed{\frac{\text{final area} - \text{initial are}}{\text{initial area}} \times 100}$$

* Signification of Percentage Elongation and Percentage Reduction in area:

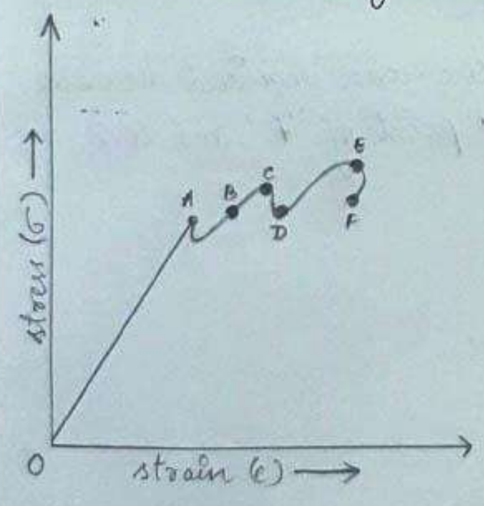
(i) Percentage elongation is a property of material which ~~prode~~ provides a value of its ductility.

(ii) The greater deformation before breaking shows the material is more ductile.

(iii) Percentage elongation is a measure of ductility.

(iv) Percentage reduction area is also a measure of ductility.

* Stress - Strain diagram for mild steel: (Imp)



→ This curve is obtained when a mild steel specimen undergoes a tensile test. The plot from O to A is a straight line, this portion obeys Hooke's law and the straight line is called the limit of proportionality.

→ In this range of extension, the stress is proportional to strain i.e. $\boxed{\sigma \propto e}$

→ If the specimen is extended beyond the limit of proportionality upto the point 'B', the material still remains elastic.

→ But from A to B, stress-strain relation is not linear.

→ The stress at 'B' is called elastic limit.

→ Again if the specimen is extended beyond the elastic limit, plastic deformation occurs.

→ In the range of 'B' to 'C' strain increases without increase in stress.

→ At point 'C' the material goes extended with decrease in load and the stress at point C is called upper-yield point.

→ At point 'D' the material again offers resistance to greater extension and the stress corresponding to this point is called lower-yield point.

→ As the load is increased, the extension increases and the point 'E' indicates the necking of the specimen and the stress corresponding to this point is called ultimate tensile stress.

→ As the extension is increased, the load required decreases and the specimen breaks at the point of 'F' and this point is called stress failure.

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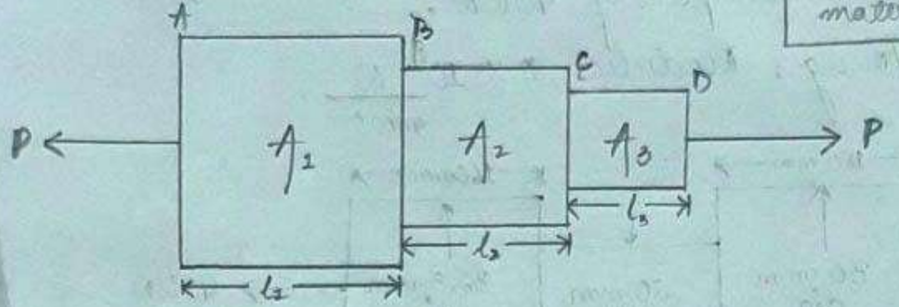
* Elongation in bar of varying section:

changes in cross section

E is same, material to material



The elongation for the portion AB : $\boxed{\Delta l_1 = \dfrac{P l_1}{A_1 E}}$

The elongation for the portion BC : $\boxed{\Delta l_2 = \dfrac{P l_2}{A_2 E}}$

The elongation for the portion CD : $\boxed{\Delta l_3 = \dfrac{P l_3}{A_3 E}}$

So total elongation : $\boxed{\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3}$

$$= \frac{P l_1}{A_1 E} + \frac{P l_2}{A_2 E} + \frac{P l_3}{A_3 E}$$

$$= \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right)$$

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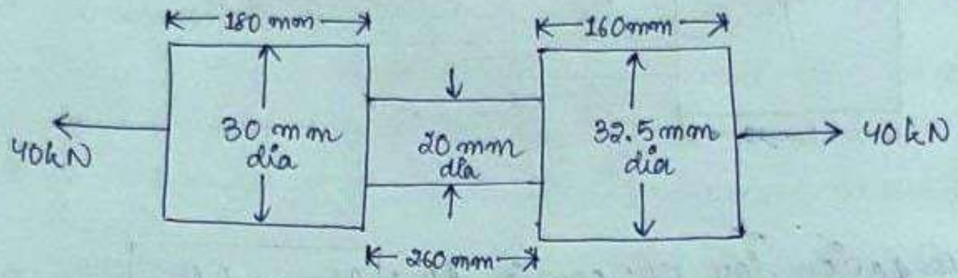
① A bar consisting of 3 lengths. Find the stresses in the three parts and the total elongation of the bar for an axial pull of 40kN.
Take Young's Modulus $2 \times 10^5 \frac{N}{mm^2}$.



→ given: $P = 40kN = 40 \times 10^3 N$

$E = 2 \times 10^5 \frac{N}{mm^2}$

$A_1 = \frac{\pi}{4}(30)^2 = 706.85 \ mm^2$.

$A_2 = \frac{\pi}{4}(20)^2 = 314.15 \ mm^2$.

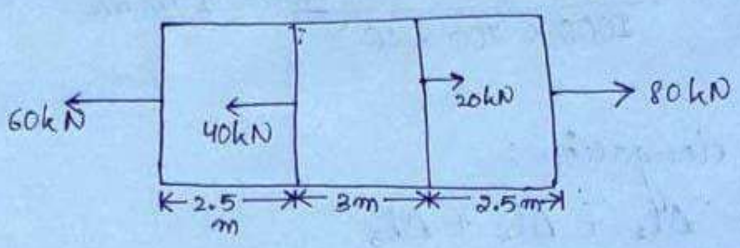
$A_3 = \frac{\pi}{4}(32.5)^2 = 829.579 \ mm^2$.

$\Delta l_1 = \frac{40 \times 10^3 \times 180}{706.85 \times 2 \times 10^5} = 0.50 \ mm$

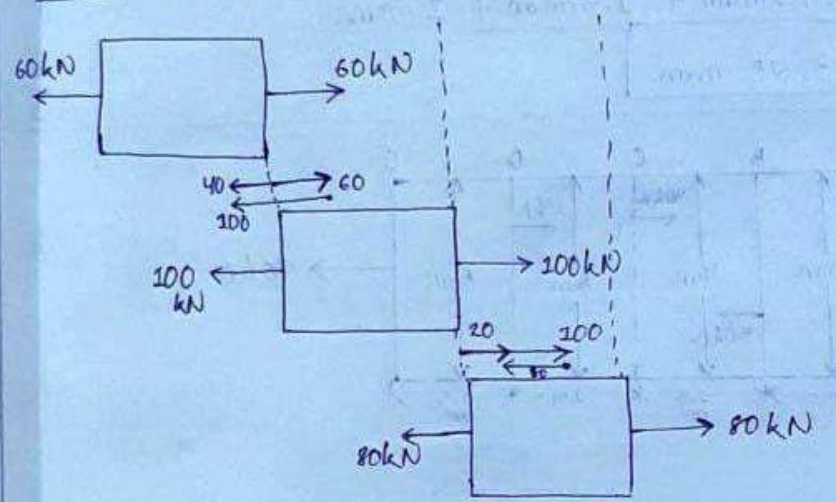
$\Delta l_2 = \frac{40 \times 10^3 \times 260}{314.15 \times 2 \times 10^5} = 0.165 \ mm$

$\Delta l_3 = \frac{40 \times 10^3 \times 160}{829.579 \times 2 \times 10^5} = 0.038 \ mm$

③ A steel rod ABCD of 1000 mm^2 crosssectional area and 8m long is subjected to forces as shown in the figure ~~bto~~ below. Find the total deformation if young's modulus of the bar is 200 GPa. All forces ~~an~~ are in kN.



FBD:



$P_1 = 60kN = 60 \times 10^3 N$

$P_2 = 100kN = 100 \times 10^3 N$

$P_3 = 80kN = 80 \times 10^3 N$

$l_1 = 2.5m = 2.5 \times 10^3 mm$

$l_2 = 3m = 3 \times 10^3 mm$

$l_3 = 2.5m = 2.5 \times 10^3 mm$

$E = 200 GPa$ $\qquad A = 1000 mm^2$

$= 200 \times 10^9 \frac{N}{m^2}$

$= 200 \times 10^9 \frac{N}{10^6 mm^2}$

$= 200 \times 10^{9-6} \frac{N}{mm^2}$

$= 200 \times 10^3 \frac{N}{mm^2}$.

$$\Delta l_1 = \frac{P_1 l_1}{AE} = \frac{60 \times 10^3 \times 2.5 \times 10^3}{1000 \times 200 \times 10^3} = 0.75 \text{ mm}$$

$$\Delta l_2 = \frac{P_2 l_2}{AE} = \frac{100 \times 10^3 \times 3 \times 10^3}{1000 \times 200 \times 10^3} = 1.5 \text{ mm}$$

$$\Delta l_3 = \frac{P_3 l_3}{AE} = \frac{80 \times 10^3 \times 2.5 \times 10^3}{1000 \times 200 \times 10^3} = 1 \text{ mm}$$

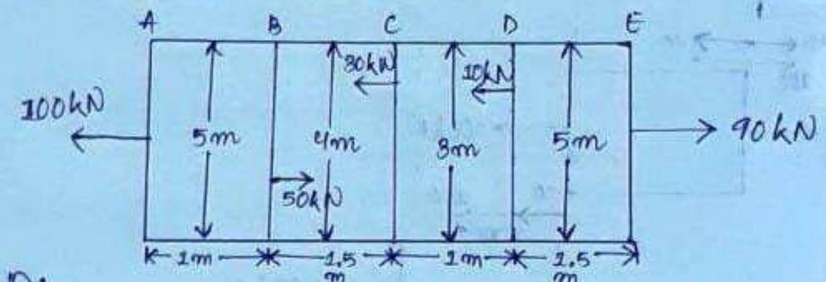
∴ Total elongation:

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$$

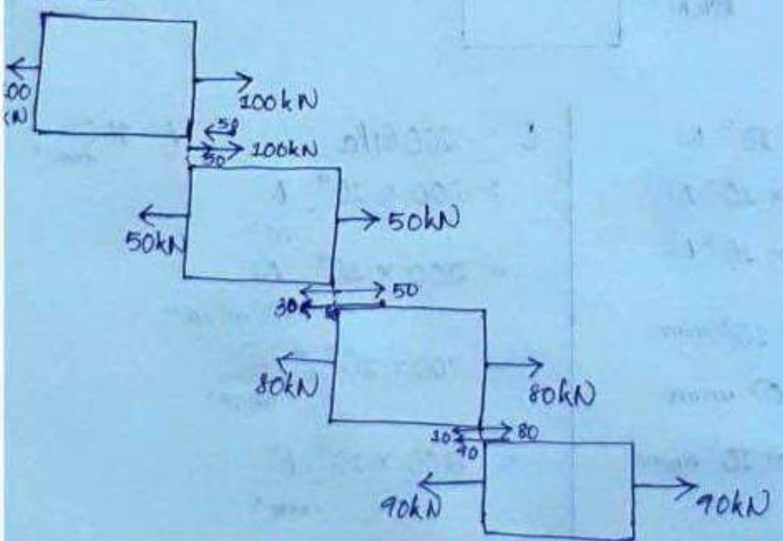
$$\Rightarrow \Delta l = 0.75\text{mm} + 1.5\text{mm} + 1\text{ mm}$$

$$\Rightarrow \boxed{\Delta l = 3.25 \text{ mm}}$$

(4)



FBD:



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$P_1 = 100kN = 100 \times 10^3 N$ | $l_1 = 1 m$ | $d_1 = 5 m$

$P_2 = 50kN = 50 \times 10^3 N$ | $l_2 = 1.5 m$ | $d_2 = 4 m$

$P_3 = 80kN = 80 \times 10^3 N$ | $l_3 = 1 m$ | $d_3 = 3 m$

$P_4 = 90kN = 90 \times 10^3 N$ | $l_4 = 1.5 m$ | $d_4 = 5 m$

$A_1 = \frac{\pi}{4}(5)^2 = 19.63 m^2$

$A_2 = \frac{\pi}{4}(4)^2 = 12.56 m^2$

$A_3 = \frac{\pi}{4}(3)^2 = 7.06 m^2$

$A_4 = \frac{\pi}{4}(5)^2 = 19.63 m^2$

$E = 2.5 MPa$
$= 2.5 \times 10^6 \frac{N}{m^2}$.

$$\Delta l_1 = \frac{P_1 l_1}{A_1 E} = \frac{100 \times 10^3 \times 1}{19.63 \times 2.5 \times 10^6} = 2.037 \times 10^{-3} m$$

$$\Delta l_2 = \frac{P_2 l_2}{A_2 E} = \frac{50 \times 10^3 \times 1.5}{12.56 \times 2.5 \times 10^6} = 2.38 \times 10^{-3} m$$

$$\Delta l_3 = \frac{P_3 l_3}{A_3 E} = \frac{80 \times 10^3 \times 1}{7.06 \times 2.5 \times 10^6} = 4.53 \times 10^{-3} m$$

$$\Delta l_4 = \frac{P_4 l_4}{A_4 E} = \frac{90 \times 10^3 \times 1.5}{19.63 \times 2.5 \times 10^6} = 2.75 \times 10^{-3} m$$

$\therefore$ Total elongation:

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_4$$

$$\Rightarrow \Delta l = (2.037 \times 10^{-3}) + (2.38 \times 10^{-3}) + 2(4.53 \times 10^{-3}) + (2.75 \times 10^{-3}) m$$

$$\Rightarrow \Delta l = 0.011697 m$$

$$\Rightarrow \boxed{\Delta l = 11.697 mm}$$

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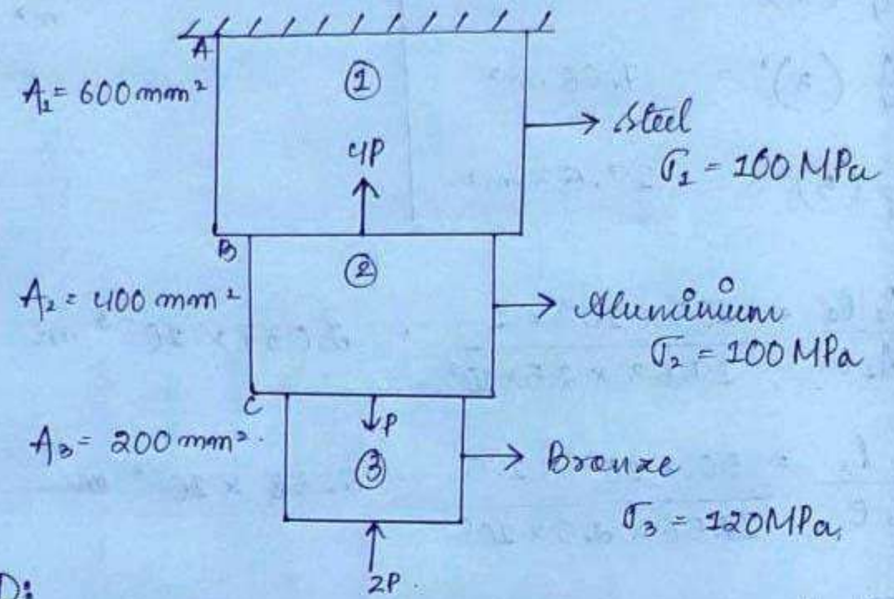
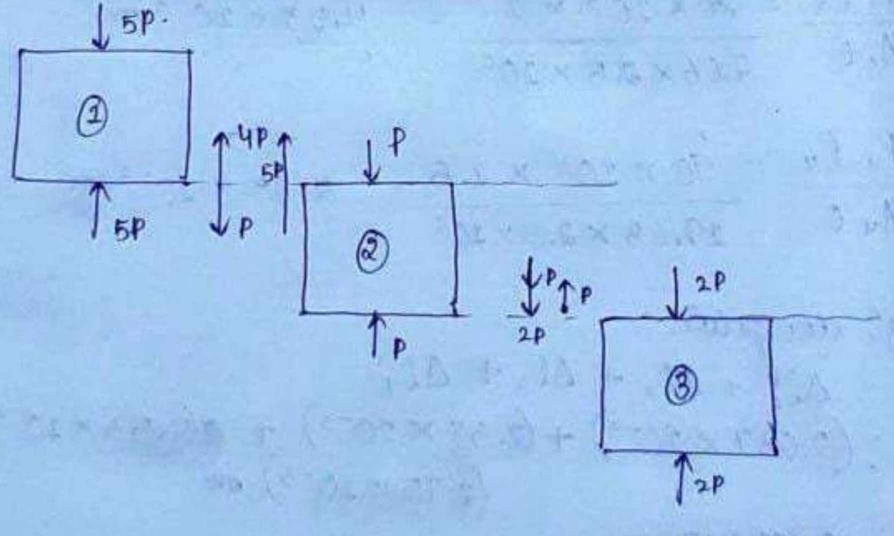
⑤ An aluminium bar is rigidly attached between steel bar and a bronze bar shown in figure. Axial forces are applied at the position indicated. Determine the maximum value of P that will not exceed stress of 160MPa in steel, 100 MPa in aluminium and 120MPa in bronze.



FBD:



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$\sigma_1 = 160\ MPa = 160 \times 10^6 \frac{N}{m^2} = 160 \times \cancel{10^6} \frac{N}{\cancel{10^6} mm^2}$

$\Rightarrow \boxed{\sigma_1 = 160 \frac{N}{mm^2}}$

$\sigma_2 = 100\ MPa = 100 \times 10^6 \frac{N}{m^2} = 100 \times \cancel{10^6} \frac{N}{\cancel{10^6} mm^2}$

$\Rightarrow \boxed{\sigma_2 = 100 \frac{N}{mm^2}}$.

$\sigma_3 = 120\ MPa = 120 \times 10^6 \frac{N}{m^2} = 120 \times \cancel{10^6} \frac{N}{\cancel{10^6} mm^2}$

$\Rightarrow \boxed{\sigma_3 = 120 \frac{N}{mm^2}}$

$A_1 = 600\ mm^2 \qquad P_1 = 5P$

$A_2 = 400\ mm^2 \qquad P_2 = P$

$A_3 = 200\ mm^2 \qquad P_3 = 2P$.

∴ we know that :

For section ① :

$$\sigma_1 = \frac{P_1}{A_1}$$

$\Rightarrow 160 = \frac{5P}{600} \Rightarrow P_1 = \frac{160 \times 600}{5}$

$\Rightarrow \boxed{P_1 = 19200\ N}$

for section ② :

$$\sigma_2 = \frac{P_2}{A_2}$$

$$\Rightarrow 200 = \frac{P}{400} \Rightarrow P_2 = 100 \times 400$$

$$\Rightarrow \boxed{P_2 = 40000 \text{ N}}$$

for section ③ :

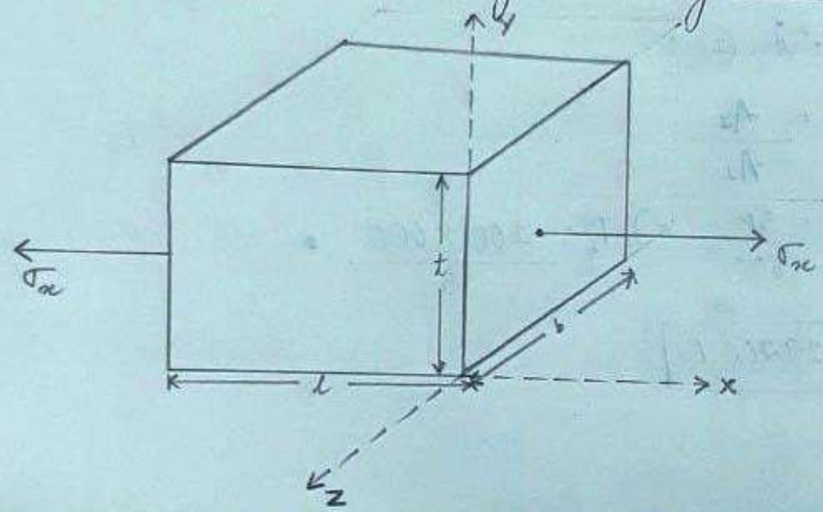
$$\sigma_3 = \frac{P_3}{A_3}$$

$$\Rightarrow 120 = \frac{2P}{200} \Rightarrow P_3 = \frac{120 \times 200}{2}$$

$$\Rightarrow \boxed{P_3 = 12000 \text{ N}}$$

27.11.2021

① Volumetric strain of a Rectangular Section Under the action of a stress along the longitudinal strain:



let l, b, t are the length, breadth and thickness of the rectangular block.

σ_x = tensile stress in x-direction

E = Young's Modulus

μ = Poisson's ratio

$= \dfrac{-\text{ latural strain}}{\text{longitudinal strain}}$

<u>longitudinal strain</u> : $\boxed{e_x = \dfrac{\sigma_x}{E}}$

$$\mu = -\frac{e_y}{e_x}$$

$$\Rightarrow e_y = -\mu \cdot e_x$$

$$\Rightarrow \boxed{e_y = -\mu \frac{\sigma_x}{E}} \qquad \left[\because e_x = \frac{\sigma_x}{E}\right]$$

Similarly, $\boxed{e_z = -\mu \dfrac{\sigma_x}{E}}$

But we know that,

$$\boxed{e_x = \frac{\delta l}{l}}, \quad \boxed{e_y = \frac{\delta b}{b}}, \quad \boxed{e_z = \frac{\delta t}{t}}$$

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we know that,

volume of a rectangular block (3D) is given by:

$$\boxed{V = l \times b \times t}$$

Now, derivating both the sides:

$$\boxed{\frac{dv}{v} = \frac{dl}{l} + \frac{db}{b} + \frac{dt}{t}}$$

$$\left[\because \frac{d}{l}(l \times b \times t) + \frac{d}{b}(l \times b \times t) + \frac{d}{t}(l \times b \times t) = \frac{dl}{l} + \frac{db}{b} + \frac{dt}{t}.\right]$$

$$\Rightarrow \boxed{e_v = e_x + e_y + e_z}$$

$$\left[\begin{aligned} &\because e_v = \frac{dv}{v} = \text{volumetric strain} \\ &e_x = \frac{dl}{l} \qquad e_z = \frac{dt}{t} \\ &e_y = \frac{db}{b} \end{aligned}\right]$$

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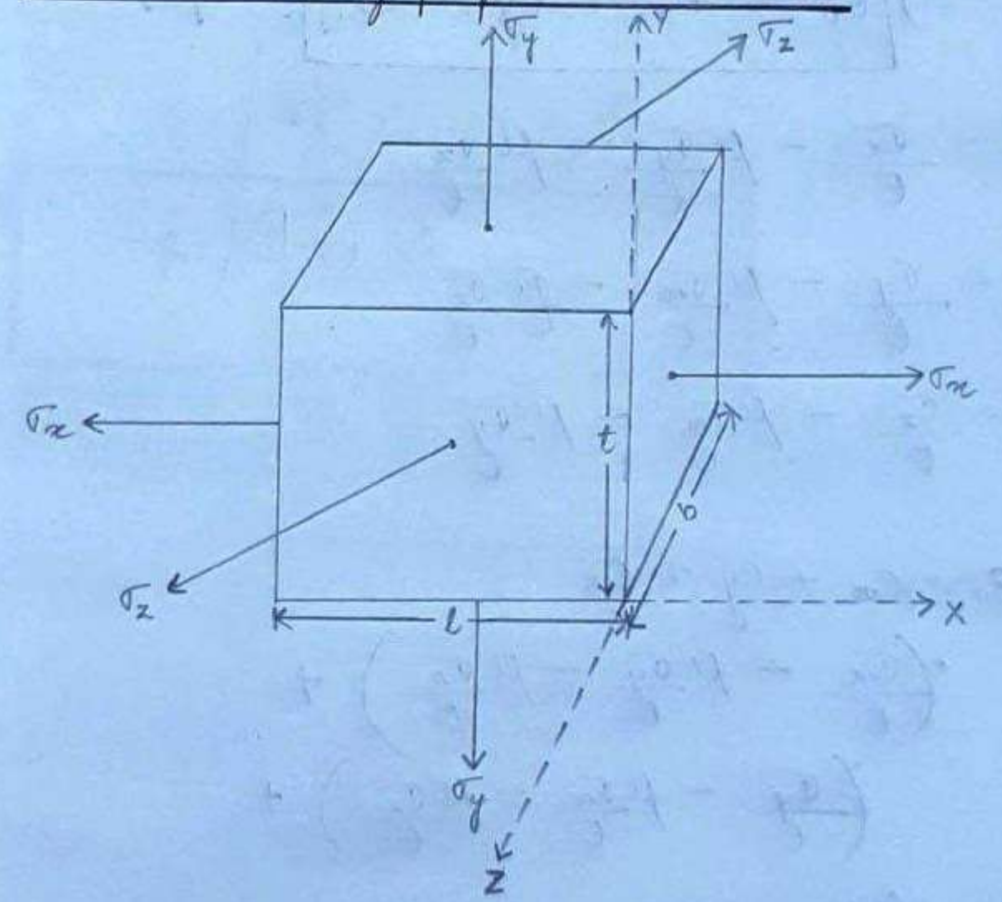
Now put the value of e_x, e_y and e_z

$$\boxed{e_v = \frac{\sigma_x}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_x}{E}}$$

$$\Rightarrow e_v = \frac{\sigma_x}{E} - 2\mu\frac{\sigma_x}{E}$$

$$\Rightarrow \boxed{e_v = \frac{\sigma_x}{E}\left(1-2\mu\right)}$$ ★★★★★

② Volumetric strain of a rectangular block subjected to three mutually perpendicular stress:



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Let l, b, t are the length, breadth and thickness of the rectangular block.

σ_x, σ_y, σ_z are the three stresses act in the direction of x, y, z.

We know that,

$$V = l \times b \times t$$

and
$$\frac{dv}{v} = \frac{dl}{l} + \frac{db}{b} + \frac{dt}{t}$$

$$\Rightarrow \boxed{e_v = e_x + e_y + e_z}$$

- $e_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$
- $e_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$
- $e_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$

$$\therefore e_v = e_x + e_y + e_z$$

$$= \left(\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}\right) + \left(\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}\right) + \left(\frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}\right)$$

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$$\Rightarrow e_v = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} + \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E} + \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

$$\Rightarrow e_v = \frac{1}{E}\left[\sigma_x - \mu\sigma_y - \mu\sigma_z + \sigma_y - \mu\sigma_x - \mu\sigma_z - \sigma_z - \mu\sigma_x - \mu\sigma_y\right]$$

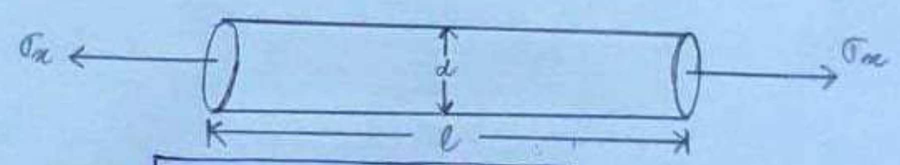
$$\Rightarrow e_v = \frac{1}{E}\left[\sigma_x - \mu\sigma_x - \mu\sigma_x + \sigma_y - \mu\sigma_y - \mu\sigma_y + \sigma_z - \mu\sigma_z - \mu\sigma_z\right]$$

$$\Rightarrow e_v = \frac{1}{E}\left[\sigma_x - 2\mu\sigma_x + \sigma_y - 2\mu\sigma_y + \sigma_z - 2\mu\sigma_z\right]$$

$$\Rightarrow e_v = \frac{1}{E}\left[\sigma_x(1-2\mu) + \sigma_y(1-2\mu) + \sigma_z(1-2\mu)\right]$$

$$\Rightarrow \boxed{e_v = \frac{1}{E}\left[(1-2\mu)(\sigma_x + \sigma_y + \sigma_z)\right]}$$

③ <u>Volumetric strain for a circular Rod:</u>



$$\boxed{e_v = 2e_d + e_l}$$

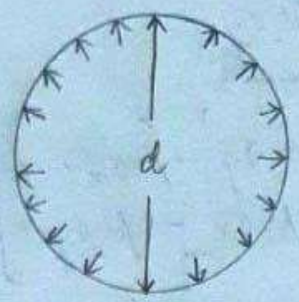
where, e_v = volumetric strain

e_d = lateral strain

e_l = longitudinal strain

Complex Stress and Strain .

(4) Volumetric strain for a sphere:

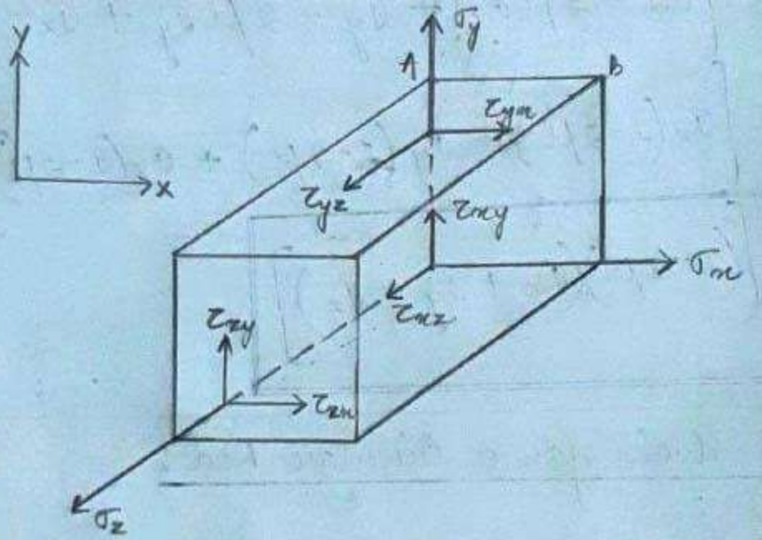


$$\boxed{e_v = 3 e_d}$$

CHAPTER - 3

29.11.2021

COMPLEX STRAIN AND STRESS :



3-D representation of stresses

(Or)

3- dimensional stress system

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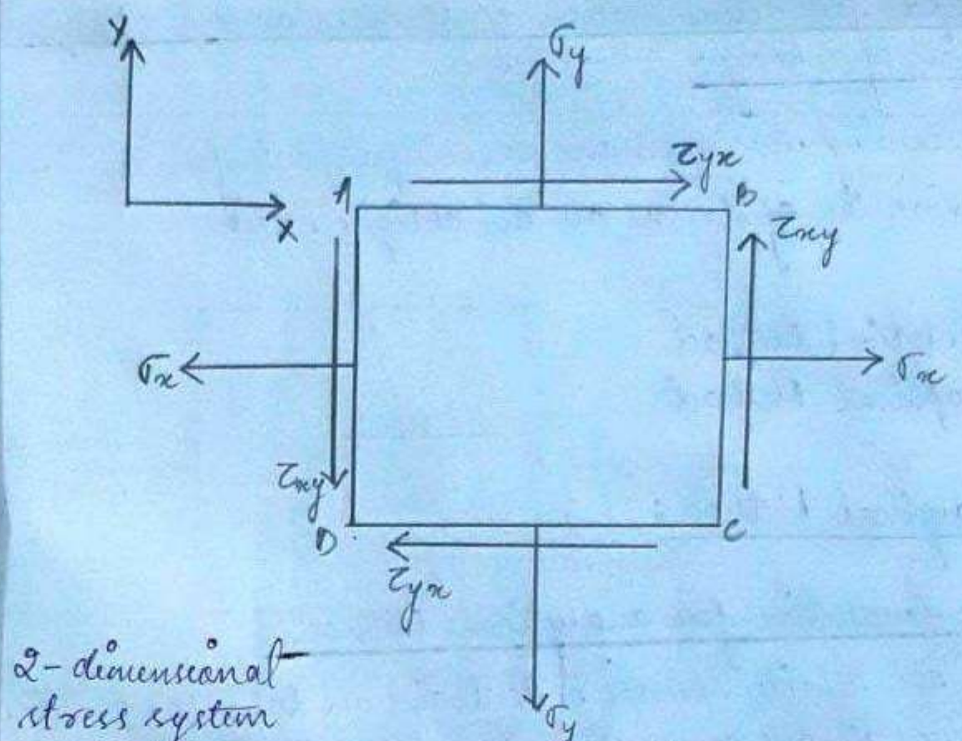

2- dimensional stress system
(or)
Plane stress condition

30.11.2021

* Principle Plane :

It has been observed that any point in a strength material there are 3 planes mutually perpendicular to each other. The plane which carry direct stresses only and half ~~on~~ no shear stresses are known as principle plane.

* Principle Stress :

The magnitude of direct stress, across a principle plane is known as principle stress.

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* Methods for determination of stresses on an oblique section of a body:

The following two methods are important for the determination of stresses on an oblique section i.e.

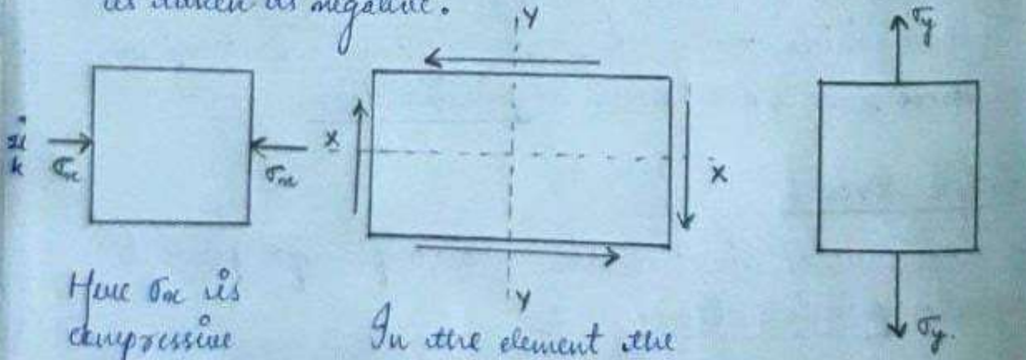
① Analytical Method
② Graphical Method

① Analytical Method:

* Sign Convention for analytical method:

ⓘ All the tensile stresses and strains are taken as positive, whereas all the compressive stresses and strains are taken as negative.

ⓘⓘ The shear stress which tends to rotate the element in the clockwise direction is taken as positive, whereas that which tends to rotate in an anticlockwise direction is taken as negative.



Here σ_x is compressive stress

In the element the shear stress in the vertical face is taken as positive, whereas the shear stress on the horizontal face is taken as negative.

Here σ_y is tensile stress

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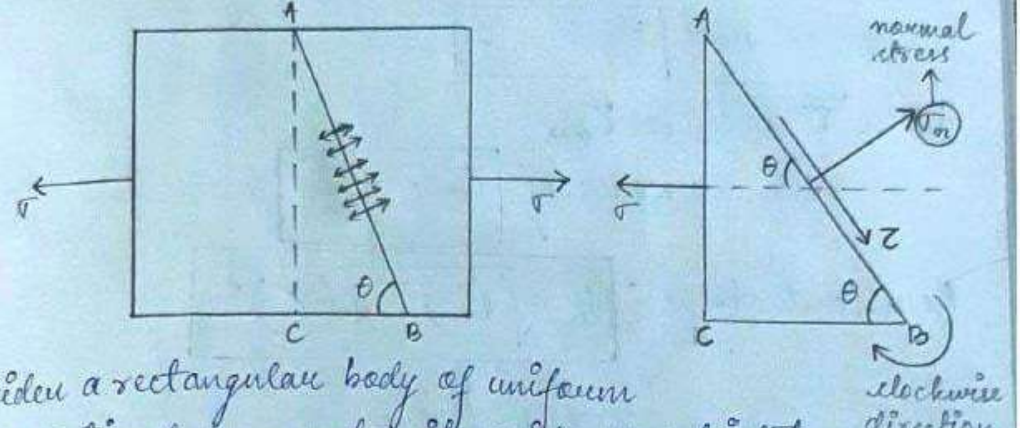
① CASE-1

Stresses on an oblique section of a body subjected to a direct stress in one plane.



Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to a direct tensile stress along X-X axis.

Now consider an oblique section AB inclined with the X-X axis.

let σ = tensile stress across the face AC

θ = angle, which the oblique section AB makes with BC i.e. with the X-X axis in clockwise direction.

The normal stress across section AB is given by:

$$\boxed{\sigma_n = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\theta}$$

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Shear Stress across the section AB,

$$\boxed{\tau = \frac{\sigma}{2}\sin 2\theta}$$

when $\theta = 45°$, then $2\theta = 90°$

and $\boxed{\sin\theta = 1}$

then 'τ' becomes τ_{max}

and the value of $\boxed{\tau_{max} = \frac{\sigma}{2}}$

Resultant stress : $\boxed{\sigma_R = \sqrt{(\sigma_n)^2 + (\tau)^2}}$

resultant of normal stress and shear stress

Questions:

(1) A wooden bar is subjected to tensile stress of 5MPa. What will be the values of normal and shear stresses across a section, which makes an angle of 25° with the direction of the tensile stresses?

→ given: $\sigma = 5\,MPa$

$\sigma_n = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\theta$

$\Rightarrow \sigma_n = \frac{5}{2} - \frac{5}{2}\cos(2\times 25)$

$\Rightarrow \sigma_n = 0.89\,\frac{N}{mm^2}$

$\Rightarrow \boxed{\sigma_n = 0.89\,MPa}$

$\tau = \frac{\sigma}{2}\sin 2\theta$

$\Rightarrow \tau = \frac{5}{2}\sin(2\times 25)$

$\Rightarrow \tau = 1.915\,\frac{N}{mm^2}$

$\Rightarrow \boxed{\tau = 1.915\,MPa}$

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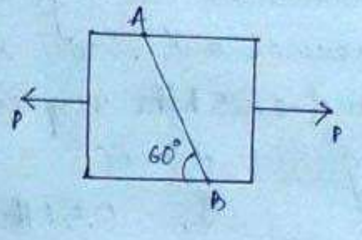
(2) Two wooden pieces 100 mm × 100 mm in cross-section are joined together along a line AB. Find the maximum force P, which can be applied if the shear stresses along the joint AB is 1.3 MPa.

→ given: $\tau = 1.3$ MPa

$P = ?$

$\theta = 60^\circ$



$$\tau = \frac{\sigma}{2} \sin(2\times60^\circ)$$

$$\Rightarrow 1.3 = \frac{\sigma}{2} \sin(2\times60^\circ)$$

$$\Rightarrow \sigma = \frac{1.3}{\sin(2\times60^\circ)} \times 2 \Rightarrow \sigma = 3.002 \frac{N}{mm^2}$$

$$\Rightarrow \boxed{\sigma = 3.002 \text{ MPa}}$$

$$\sigma = \frac{P}{A}$$

$$\Rightarrow 3.002 = \frac{P}{100\times100}$$

$$\Rightarrow P = 3.02 \times 100 \times 100$$

$$\Rightarrow \boxed{P = 30020 N}$$

$$\Rightarrow P = \frac{30020}{10^3} kN \Rightarrow \boxed{P = 30.02 \text{ kN}}$$

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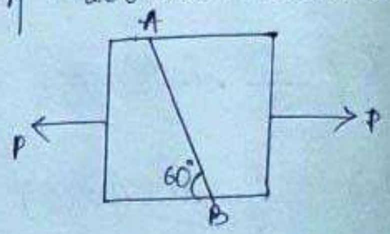
③ A tension member is formed by connecting two wooden members 200 mm × 100 mm as shown in the figure:

Determine the safe value (minimum value.) of the force P if permeable normal and shear stresses in the joints are 0.5 MPa and 1.25 MPa respectively.

→ given: $\theta = 60^\circ$ $\quad A = 200\,mm \times 100\,mm$

$\sigma_n = 0.5\,MPa$

$\tau = 1.25\,MPa$



$$\tau = \frac{\sigma}{2}\sin(2\times 60)$$

$$\Rightarrow 1.25 = \frac{\sigma}{2}\sin(2\times 60)$$

$$\Rightarrow \sigma = \frac{1.25}{\sin(2\times 60)} \times 2.$$

$$\Rightarrow \boxed{\sigma = 2.88\,MPa}$$

$$\sigma = \frac{P}{A}$$

$$\Rightarrow 2.88 = \frac{P}{200\times 100}$$

$$\Rightarrow P = 2.88 \times 200 \times 100$$

$$\Rightarrow \boxed{P = 57600\,N}$$

$$\Rightarrow P = \frac{57600}{10^3}\,kN \Rightarrow \boxed{P = 57.6\,kN}$$

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$$\sigma_n = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\theta$$

$$\Rightarrow 0.5 = \frac{\sigma}{2}\left(1 - \cos 2\theta\right)$$

$$\Rightarrow 0.5 = \frac{\sigma}{2}\left(1 - \cos(2\times 60)\right)$$

$$\Rightarrow \sigma = \frac{0.5}{\left(1 - \cos(2\times 60)\right)} \times 2$$

$$\Rightarrow \boxed{\sigma = 0.67\,MPa}$$

$$\sigma = \frac{P}{A}$$

$$\Rightarrow 0.67 = \frac{P}{200 \times 100}$$

$$\Rightarrow P = 0.67 \times 200 \times 100$$

$$\Rightarrow \boxed{P = 13400\,N}$$

$$\Rightarrow P = \frac{13400}{10^3}\,kN$$

$$\Rightarrow \boxed{P = 13.4\,kN}$$

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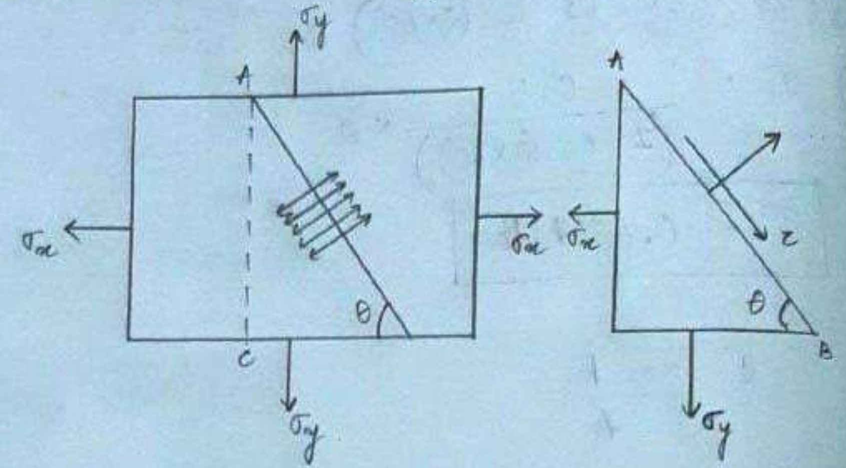
② CASE-2 (Imp)

Stresses on an oblique section of a body subjected to direct stresses in 2 mutually perpendicular directions:



Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to direct tensile stresses in two mutually perpendicular directions along X-X axis and Y-Y axis.

Now, let us consider an oblique section AB inclined with X-X axis.

σ_x = tensile stress along X-X axis (major tensile stress)

σ_y = tensile stress along Y-Y axis (minor tensile stress)

θ = angle, which the oblique section AB makes with X-X axis in clockwise direction.

normal stress: 01. 12. 2021

$$\boxed{\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta}$$

shear stress: $$\boxed{\tau = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta}$$

τ_{max} : when $\theta = 45$

$2\theta = 90$

$\sin 90^\circ = 1$

$$\boxed{\tau_{max} = \frac{\sigma_x - \sigma_y}{2}}$$

$$\boxed{\sigma_R = \sqrt{(\sigma_n)^2 + (\tau)^2}}$$ (σ_R - resultant stress)

Direction of resultant stress:

$$\boxed{\tan \alpha = \frac{\tau}{\sigma_n}}$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{\tau}{\sigma_n}\right)}$$

Questions:

① The stresses at a point in a component are 100 MPa (tensile) and 50 MPa (compressive).
Determine the magnitude of the normal and shear stresses on a plane inclined at an angle of 25° with tensile stress.
Also determine the direction of the resultant stress and the magnitude of the maximum intensity of shear stress.

→ given: $\sigma_x = 100\ MPa$ (tensile) $\sigma_y = 50\ MPa$ (compressive) $= -50$

$\theta = 25°$

$$\therefore \sigma_n = \left(\frac{100 + (-50)}{2}\right) - \left(\frac{100 - (-50)}{2}\right) \cos(2 \times 25)$$

$$\Rightarrow \sigma_n = \left(\frac{100 - 50}{2}\right) - \left(\frac{100 + 50}{2}\right) \cos(2 \times 25)$$

$$\Rightarrow \boxed{\sigma_n = -23.20\ MPa}$$

$$\tau = \left(\frac{100 - (-50)}{2}\right) \sin(2 \times 25)$$

$$\Rightarrow \tau = \left(\frac{100 + 50}{2}\right) \sin(2 \times 25)$$

$$\Rightarrow \boxed{\tau = 57.45\ MPa}$$

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$$\sigma_R = \sqrt{(\sigma_n)^2 + (\tau)^2}$$

$$\Rightarrow \sigma_R = \sqrt{(-23.20)^2 + (57.45)^2}$$

$$\Rightarrow \boxed{\sigma_R = 61.95 \text{ MPa}}$$

$$\alpha = \tan^{-1}\left(\frac{\tau}{\sigma_n}\right)$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{57.45}{-23.20}\right)$$

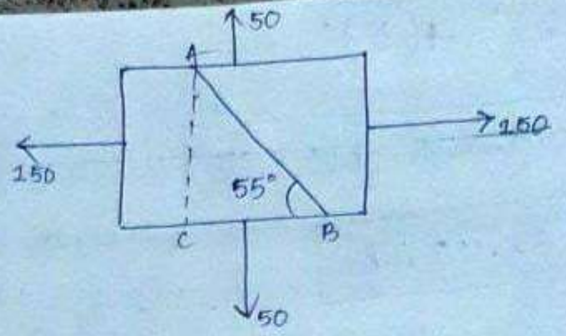
$$\Rightarrow \boxed{\alpha = \tan^{-1}(-68.009)}$$

$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2} = \frac{100-(-50)}{2}$$

$$\Rightarrow \boxed{\tau_{max} = 75 \text{ MPa}}$$

(2) The stresses at a point of a machine component are 150 MPa and 50 MPa both tensile. Find the intensities of normal, shear and resultant stresses on a plane inclined at an angle of 55° with the axis of major tensile stress. (X-X axis)
Also find the magnitude of the maximum shear stress in the component.

→ given: $\sigma_x = 150 \text{ MPa} (+)$ $\theta = 55°$
$\sigma_y = 50 \text{ MPa} (+)$



$$\sigma_n = \left(\frac{150+50}{2}\right) - \left(\frac{150-50}{2}\right)\cos(2\times 55)$$

$$\Rightarrow \boxed{\sigma_n = 117.10\ \text{MPa}}$$

$$\tau = \left(\frac{150-50}{2}\right)\sin(2\times 55)$$

$$\Rightarrow \boxed{\tau = 46.98\ \text{MPa}}$$

$$\sigma_R = \sqrt{(117.10)^2 + (46.98)^2}$$

$$\Rightarrow \boxed{\sigma_R = 126.17\ \text{MPa}}$$

$$\tau_{max} = \frac{150-50}{2} \Rightarrow \boxed{\tau_{max} = 50\ \text{MPa}}$$

$$\alpha = \tan^{-1}\left(\frac{46.98}{117.10}\right)$$

$$\Rightarrow \boxed{\alpha = \tan^{-1}(21.86)}$$

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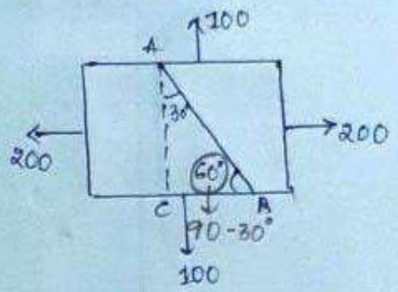
③ A point in a strained material is subjected to two mutually perpendicular tensile stresses of 200 MPa and 100 MPa. Determine the intensities of normal, shear and resultant stresses on a plane inclined at 30° with the axis of minor tensile stress. (Y-Y axis)

Imp

→ given: $\sigma_x = 200$ MPa (+)

$\sigma_y = 100$ MPa (+)



angle with the minor tensile stress = 30°

angle with the major tensile stress = 90° − 30° = 60°

$$\sigma_n = \left(\frac{200+100}{2}\right) - \left(\frac{200-100}{2}\right)\cos(2\times 60)$$

$$\Rightarrow \boxed{\sigma_n = 175\ \text{MPa}}$$

$$\tau = \left(\frac{200-100}{2}\right)\sin(2\times 60)$$

$$\Rightarrow \boxed{\tau = 43.30\ \text{MPa}}$$

$$\sigma_R = \sqrt{(175)^2 + (43.30)^2} \Rightarrow \boxed{\sigma_R = 180.27\ \text{MPa}}$$

$$\tau_{max} = \frac{200-100}{2} \Rightarrow \boxed{\tau_{max} = 50\ \text{MPa}}$$

$$\alpha = \tan^{-1}\left(\frac{43.30}{175}\right) \Rightarrow \boxed{\alpha = \tan^{-1}(13.89)}$$

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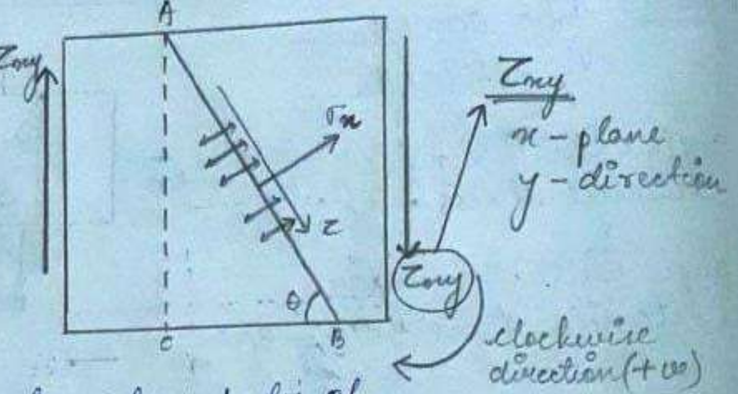
(3) CASE-3 (Imp)

Stresses on an oblique section of a body subjected to a simple shear stress only.



Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to a positive (clockwise) shear stress along X-X axis.

Let us consider an oblique section AB inclined with X-X axis on which we are required to find out the axis.

Let, τ_{xy} = positive shear stress along X-X axis

θ = angle, which the oblique section AB makes with X-X axis.

normal stress: $\boxed{\sigma_n = \tau_{xy} \sin 2\theta}$

shear stress: $\boxed{\tau = -\tau_{xy} \cos 2\theta}$

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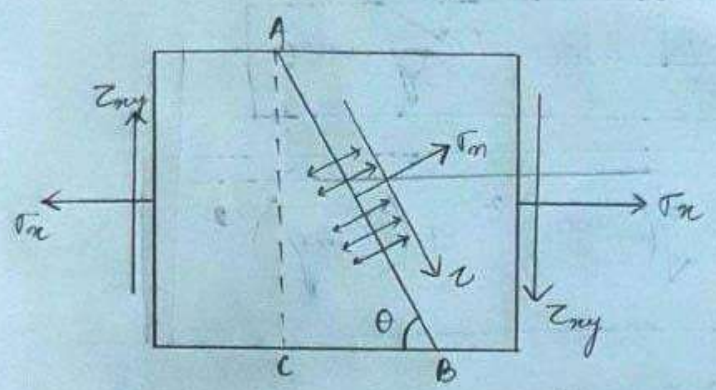
Resultant stress: $\boxed{\sigma_R = \sqrt{(\sigma_n)^2 + (\tau)^2}}$

(4) **CASE-4 (Imp)** (Combo of Case 1 and Case-3)

Stresses on an oblique section of a body subjected to direct stress in one plane accompanied by a simple shear stress.



Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to tensile stress along X-X axis accompanied with a positive (clockwise) shear stress along X-X axis.

Now let us consider an oblique section AB inclined with X-X axis on which we are required to find out the stresses.

Let, σ_x = tensile stress along X-X axis

τ_{xy} = positive (clockwise) shear stress along X-X axis

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θ = angle, which the oblique section AB makes with X–X axis in clockwise direction.

normal stress:

$$\sigma_n = \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

shear stress:

$$\tau = \frac{\sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

Maximum Principle Stress:

$$\sigma_{p_1} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 - \left(\tau_{xy}\right)^2}$$

Minimum Principle Stress:

$$\sigma_{p_2} = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 - \left(\tau_{xy}\right)^2}$$

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Questions:

(1) A plane element in a body is subjected to the tensile stress of 100 MPa accompanied with a shear stress of 25 MPa.

Find : (i) the normal and shear stress on a plane inclined at an angle of 20° with the tensile stress.

(ii) Maximum shear stress on the plane.

→ given: $\theta = 20^\circ$

$\sigma_x = 100\ \text{MPa}$

$\tau_{xy} = 25\ \text{MPa}$

(i) normal stress:

$$\sigma_n = \frac{100}{2} - \frac{100}{2}\cos(2\times 20) - 25\sin(2\times 20)$$

$$\Rightarrow \boxed{\sigma_n = -4.37\ \text{MPa}}$$

shear stress:

$$\tau = \frac{100}{2}\sin(2\times 20) - 25\cos(2\times 20)$$

$$\Rightarrow \boxed{\tau = 12.98\ \text{MPa}}$$

(ii) $$\tau_{max} = \sqrt{\left(\frac{100}{2}\right)^2 + (25)^2}$$

$$\Rightarrow \boxed{\tau_{max} = 55.90\ \text{MPa}}$$

② An element in a strain body is subjected to a compressive stress 200MPa and a clockwise shear stress of 50MPa on the same plane.
Calculate the values of normal and shear stresses on a plane inclined at 35° with the compressive stress.
Also calculate the value of maximum shear stress in the element.

→ given: $\sigma_{xy} = 200\ \text{MPa}$ (compressive)

$= (-200)\ \text{MPa}$

$\tau_{x} = 50\ \text{MPa}$

$\theta = 35°$

$$\sigma_n = \left(\frac{-200}{2}\right) - \left(\frac{-200}{2}\right)\cos(2\times35) - 50\sin(2\times35)$$

$$\Rightarrow \boxed{\sigma_n = -112.78\ \text{MPa}}$$

$$\tau = \frac{-200}{2}\sin(2\times35) - 50\cos(2\times35)$$

$$\Rightarrow \boxed{\tau = -111.07\ \text{MPa}}$$

$$\tau_{max} = \sqrt{\left(\frac{-200}{2}\right)^2 + (50)^2}$$

$$\Rightarrow \boxed{\tau_{max} = 111.80\ \text{MPa}}$$

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③ An element in a strained body is subjected to tensile stress of 150 MPa and a shear stress of 50 MPa tending to rotate the element in an anticlockwise direction.

<u>Find</u>: ⓘ the magnitude of the normal and shear stresses on a section inclined at 40° with the tensile stress.

ⓘⓘ the magnitude of the ~~shear~~ maximum shear stress that can exist on the element.

→ <u>given</u>: $\sigma_x = 150\ MPa$ $\theta = 40°$

$\tau_{xy} = 50\ MPa$ (in anticlockwise direction)

$= (-50\ MPa)$

$$\sigma_n = \frac{150}{2} - \frac{150}{2}\cos(2\times 40) - (-50)\sin(2\times 40)$$

$$\Rightarrow \boxed{\sigma_n = 111.21\ MPa}$$

$$\tau = \frac{150}{2}\sin(2\times 40) - (-50)\cos(2\times 40)$$

$$\Rightarrow \boxed{\tau = 82.54\ MPa}$$

$$\tau_{max} = \sqrt{\left(\frac{150}{2}\right)^2 + (-50)^2}$$

$$\boxed{\tau_{max} = 90.13\ MPa}$$

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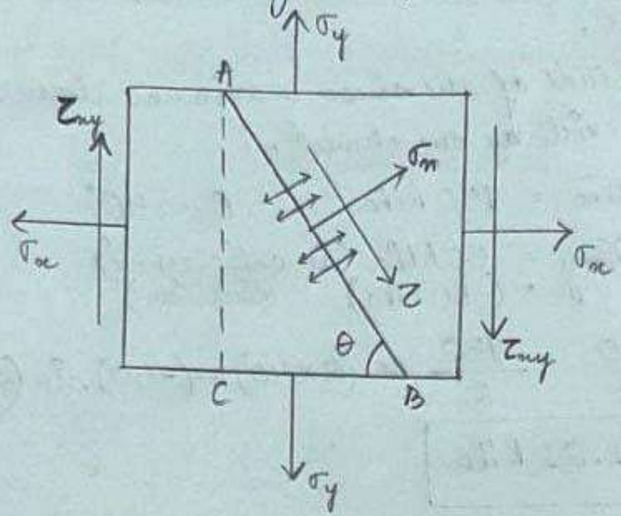
⑤ CASE - 5 ★★★★★ VVV Imp (Combo of case - 2 and case - 3)

Stresses on an oblique section of a body subjected to direct stresses in two mutually perpendicular direction accompanied by a simple shear stresses.



Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to tensile stresses along X - X axis and Y - Y axis respectively and accompanied by a positive (clockwise) shear stress along X - X axis.

Now, consider an oblique section AB inclined with X - X axis on which we are required to find out the stresses.

let, σ_x = tensile stress along X - X axis

σ_y = tensile stress along Y - Y axis

τ_{xy} = positive (clockwise) shear stress along X - X axis

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θ = angle, which the oblique section AB makes with X-X axis in an anti-clockwise direction

normal stress:

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

shear stress:

$$\tau = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta - \tau_{xy}\cos 2\theta$$

resultant stress:

$$\sigma_R = \sqrt{(\sigma_n)^2 + (\tau)^2}$$

Maximum Principle stress:

$$\sigma_{p_1} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Minimum Principle stress:

$$\sigma_{p_2} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

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maximum shear:

$$\boxed{\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}}$$

Questions:

① A point is subjected to tensile stress of 250MPa in horizontal direction and another tensile stress of 100MPa in the vertical direction. The point is also subjected to a simple ~~stresses~~ shear stress of 25MPa such that when it is associated with the major tensile stress, it tends to rotate the element in clockwise direction. What is the magnitude of normal and shear stress on the section inclined at an angle 70° with the minor tensile stress. Also find out the principle stresses.

given:) $\sigma_x = 250$ MPa

$\sigma_y = 100$ MPa

$\tau_{xy} = 25$ MPa (+ve)

angle with the minor tensile ~~axi~~ stress = 70°

angle with the major tensile stress = 90° − 70°
= 20°

$$\sigma_n = \left(\frac{250+100}{2}\right) - \left(\frac{250-100}{2}\right)\cos(2\times 20) - 25\sin(2\times 20)$$

$$\Rightarrow \boxed{\sigma_n = 101.47 \text{ MPa}}$$

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$$\tau = \left(\frac{250-100}{2}\right)\sin(2\times 20) - 25\cos(2\times 20)$$

$$\Rightarrow \boxed{\tau = 29.05\ MPa}$$

$$\sigma_R = \sqrt{(101.47)^2 + (29.05)^2}$$

$$\Rightarrow \boxed{\sigma_R = 105.54\ MPa}$$

$$\tau_{max} = \sqrt{\left(\frac{250-100}{2}\right)^2 + (25)^2}$$

$$\Rightarrow \boxed{\tau_{max} = 79.05\ MPa}$$

$$\sigma_{P_1} = \frac{250+100}{2} + \sqrt{\left(\frac{250-100}{2}\right)^2 + (25)^2}$$

$$\Rightarrow \boxed{\sigma_{P_1} = 254.05\ MPa}$$

$$\sigma_{P_2} = \frac{250-100}{2} - \sqrt{\left(\frac{250-100}{2}\right)^2 + (25)^2}$$

$$\Rightarrow \boxed{\sigma_{P_2} = 95.94\ MPa}$$

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(2) A plane ~~is~~ element is subjected to tensile stresses of 400 MPa on one plane and 150 MPa on the another plane. Each of the above stress is accompanied by a shear stress of 100 MPa such that when associated with the minor tensile stress, tends to rotate the element in anti-clockwise direction.

Imp

Find: (a) principle stresses and their direction.

(b) maximum shearing stresses and the direction of the plane on which they act.

→ given: $\sigma_x = 400\,MPa$ $\sigma_y = 150\,MPa$

$\tau_{xy} = 100\,MPa$ (anticlockwise direction) $= -100\,MPa$.

$$\sigma_{P_1} = \frac{400+150}{2} + \sqrt{\left(\frac{400-150}{2}\right)^2 + (-100)^2}$$

$$\Rightarrow \boxed{\sigma_{P_1} = 435.07\,MPa}$$

$$\sigma_{P_2} = \frac{400+150}{2} - \sqrt{\left(\frac{400-150}{2}\right)^2 + (-100)^2}$$

$$\Rightarrow \boxed{\sigma_{P_2} = 114.92\,MPa}$$

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$$\tau_{max} = \sqrt{\left(\frac{400-150}{2}\right)^2 + (-100)^2}$$

$$\Rightarrow \boxed{\tau_{max} = 160.07 \text{ MPa}}$$

* **Direction of Principle Stress:**

$$\boxed{\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

* **Direction of Maximum Shear Stress:**

$$\boxed{\tan 2\alpha = \frac{\sigma_x - \sigma_y}{2\tau_{xy}}}$$

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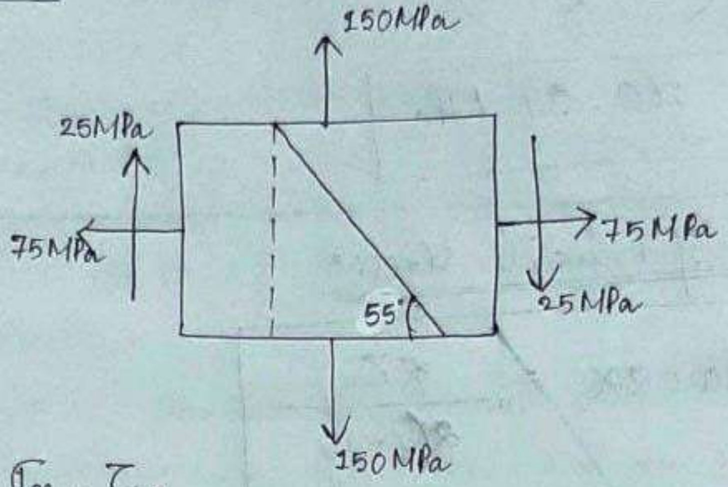
Questions:

(1)



Find σ_n, τ, τ_{max}, σ_{P_1}, σ_{P_2}.

→ given: $\sigma_x = 75\ MPa$ $\theta = 55°$

$\sigma_y = 150\ MPa$ $\tau_{xy} = 25\ MPa$

$$\sigma_n = \left(\frac{75+150}{2}\right) - \left(\frac{75-150}{2}\right)\cos(2\times 55) - 25\sin(2\times 55)$$

$$\Rightarrow \boxed{\sigma_n = 76.18\ MPa}$$

$$\tau = \left(\frac{75-150}{2}\right)\sin(2\times 55) - 25\cos(2\times 55)$$

$$\Rightarrow \boxed{\tau = -26.68\ MPa}$$

$$\tau_{max} = \sqrt{\left(\frac{75-150}{2}\right)^2 + (25)^2}$$

$$\Rightarrow \boxed{\tau_{max} = 45.06\ MPa}$$

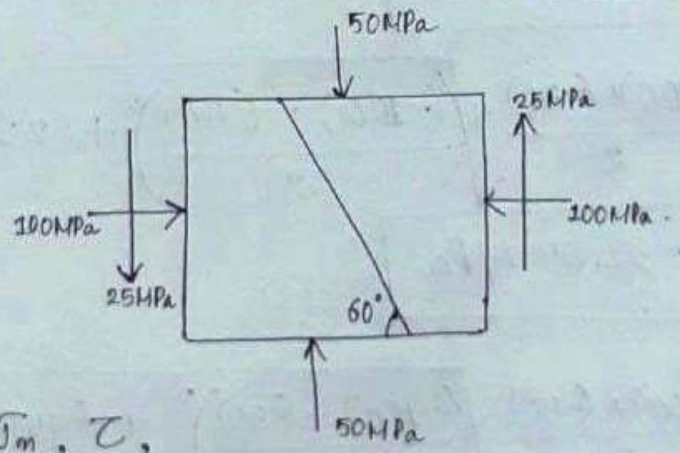
$$\sigma_{p_1} = \frac{75+150}{2} + \sqrt{\left(\frac{75-150}{2}\right)^2 + (25)^2}$$

$$\Rightarrow \boxed{\sigma_{p_1} = 157.56\ MPa}$$

$$\sigma_{p_2} = \frac{75+150}{2} - \sqrt{\left(\frac{75-150}{2}\right)^2 + (25)^2}$$

$$\Rightarrow \boxed{\sigma_{p_2} = 67.43\ MPa}$$

(2)



find σ_n, τ, τ_{max}, σ_{p_1}, σ_{p_2}

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→ given: $\sigma_x = -100\,MPa$ $\tau_{xy} = -25\,MPa$

$\sigma_y = -50\,MPa$ $\theta = 60°$

$$\sigma_n = \left(\frac{(-100)+(-50)}{2}\right) + \left(\frac{(-100)-(-50)}{2}\right) \times \cos(2\times60) - (-25)\sin(2\times60)$$

$$\Rightarrow \boxed{\sigma_n = -65.84\ MPa}$$

$$\tau = \left(\frac{(-100)-(-50)}{2}\right)\sin(2\times60) - (-25)\cos(2\times60)$$

$$\Rightarrow \boxed{\tau = -34.15\ MPa}$$

$$\sigma_{P_1} = \frac{(-100)+(-50)}{2} + \sqrt{\left(\frac{(-100)-(-50)}{2}\right)^2 + (-25)^2}$$

$$\Rightarrow \boxed{\sigma_{P_1} = -39.64\ MPa}$$

$$\sigma_{P_2} = \frac{(-100)+(-50)}{2} - \sqrt{\left(\frac{(-100)-(-50)}{2}\right)^2 + (-25)^2}$$

$$\Rightarrow \boxed{\sigma_{P_2} = -110.35\ MPa}$$

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$$\tau_{max} = \sqrt{\left(\frac{(-100) - (-50)}{2}\right)^2 + (-25)^2}$$

$$\Rightarrow \boxed{\tau_{max} = 35.35 \text{ MPa}}$$

Chapter 2

REVIEW OF BASIC CONCEPT

CG and M.I

Centroid :-

It is the Point at which the total area of the Plane figure (namely rectangle, Square, tringle, circle etc) is assumed to be Concentrated.

Centre of gravity :-

It is Point through which the resultant of the distributed gravity force (weights) act irrespecti of the orientation of the body.

Centre of mass

→ It is the Point where the entire mass of the body may be assumed to be concentrated.

→ For all Practical Purpose the Centroid and Centre of gravity are assumed to be the Same.

Moment of Inertia :-

The moment of inertia of a body about an axis is defined as the resistance offered by the body to rotation about that axis.

- it is also defined as the Product of the area and the Square of the distance of the Center of gravity of the area from that axis.
- Moment of is denoted by I.
- Hence the moment of inertia the 'x' axis is represented by I_{xx} and about the Y-axis is represented by I_{yy}.

Parallel axis Theorem :-

Parallel axis theorem states that the moment of inertia of an area with respect to any axis in its Plane is equal to the moment of inertia of the area with to a Parallel Centroidal axis Plus the Product of the area and the distance between the two axes.

Perpendicular axis theorem (Polar moment of Inertia)

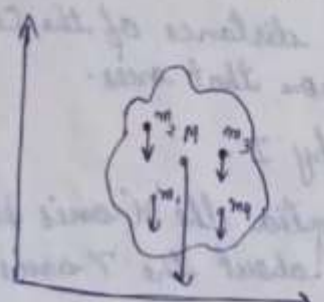
Perpendicular axis theorem States that the moment of inertia of an area with respect to an axis Perpendicular to the x-y Plane (z axis) and Passing through a Pole O is equal to the Sum of the moment of inertia of the area about the other two axis (X & y axis) Passing through Pole.

• It's also called as Polar moment of inertia and is denoted by the letter J. $J = I_{zz} = I_{xx} + I_{yy}$

Lamina :-

Lamina Ex:- Sheet of a Paper

Thin layer

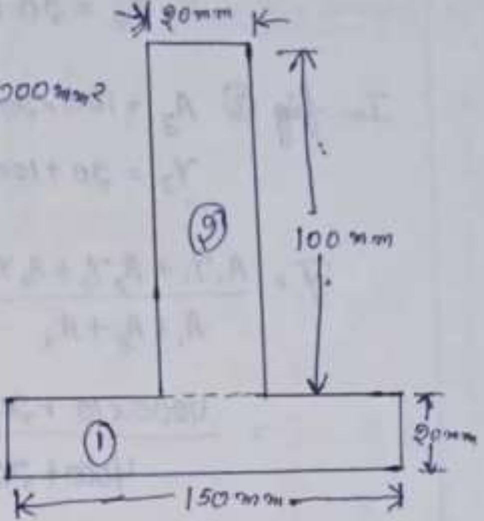


Center of gravity :-

$$\bar{X} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} , \quad \bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$\bar{X} = \frac{\Sigma a x}{\Sigma a} \qquad \bar{Y} = \frac{\Sigma a y}{\Sigma a}$$

① Find the Centroid of the lamina as shown in figure.



Ans: In fig ① $A_1 = 150 \times 20 = 3000 mm^2$

$X_1 = \frac{150}{2} = 75 mm$

$Y_1 = \frac{20}{2} = 10 mm$

In fig ②

$A_2 = 100 \times 20 = 2000 mm^2$

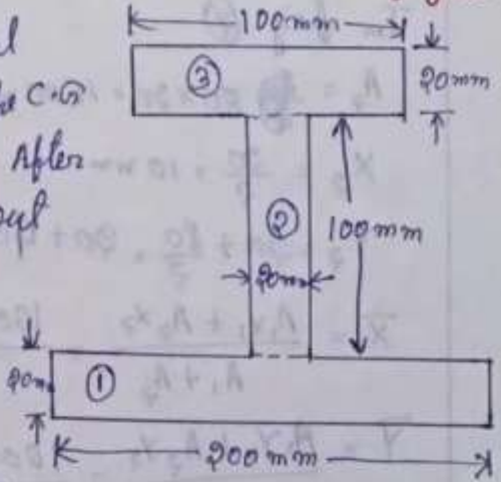
$X_2 = 65 + \frac{20}{2} = 75 mm$

$Y_2 = 20 + \frac{100}{2} = 70 mm$

As the Section Simetrycal about Y-Y axis. Then the C.G will lie on that axis. After that we have to find out $\overline{Y} = \frac{A_1Y_1 + A_2Y_2}{A_1 + A_2}$

$$\overline{Y} = \frac{A_1Y_1 + A_2Y_2}{A_1 + A_2} = \frac{3000 \times 10 + 2000 \times 70}{3000 + 2000} = 34 mm$$

② Find the centroid of the lamina as shown in figure.



Ans: As the Section Simetrycal about Y-Y axis. Then the C.G will lie on that axis. After that we have to fing out

$\overline{Y} = \frac{A_1Y_1 + A_2Y_2}{A_1 + A_2}$

In fig ①

$A_1 = 200 \times 20 = 4000 mm^2$

$Y_1 = \frac{20}{2} = 10 mm$

In fig ② $A_2 = 100 \times 20 = 2000\ mm^2$

$y_2 = 20 + \frac{100}{2} = 70\ mm$

In fig ③ $A_3 = 100 \times 20 = 2000\ mm^2$

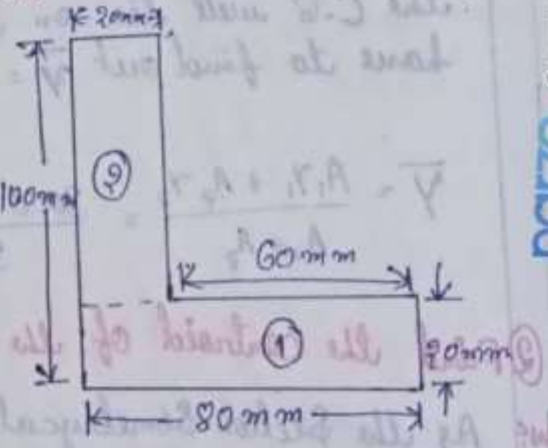
$y_3 = 20 + 100 + \frac{20}{2} = 130\ mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{4000 \times 10 + 2000 \times 70 + 2000 \times 130}{4000 + 2000 + 2000}$$

$= 55\ mm.$

Find the centroid of the lamina as shown in figure.



In fig ①

$A_1 = 80 \times 20 = 1600\ mm^2$

$X_1 = \frac{80}{2} = 40\ mm$

$y_1 = \frac{20}{2} = 10\ mm$

In fig ②

$A_2 = 80 \times 20 = 1600\ mm^2$

$X_2 = \frac{20}{2} = 10\ mm$

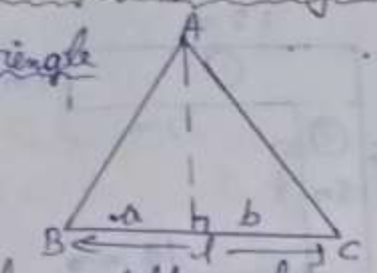
$y_2 = 20 + \frac{80}{2} = 20 + 40 = 60\ mm$

$$\bar{X} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{1600 \times 40 + 1600 \times 10}{1600 + 1600} = 25\ mm$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{1600 \times 10 + 1600 \times 60}{1600 + 1600} = 35\ mm$$

C.G For following Section :-

Triangle



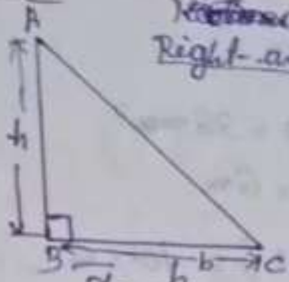
from left end

$$\bar{x} = \frac{l+a}{3}$$

form right end

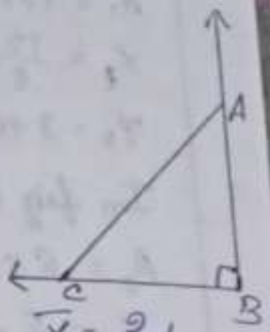
$$\bar{x} = \frac{l+b}{3}$$

~~Right~~ Right-angle-triangle



$$\bar{y} = \frac{h}{3}$$

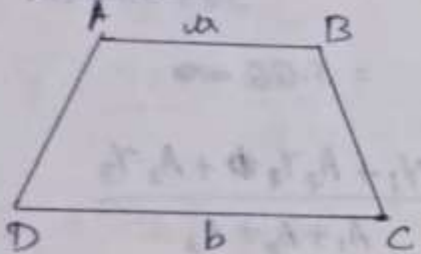
$$\bar{x} = \frac{b}{3}$$



$$\bar{x} = \frac{2}{3}b$$

$$\bar{y} = \frac{h}{3}$$

Trapizeum



~~C.G (for DC~~

$$C.G\ (\text{from } DC) = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$$

$$\text{From } AB = \left(\frac{a+2b}{a+b}\right)\frac{h}{3}$$

$$\text{Area} = \frac{1}{2}(a+b)\times h$$

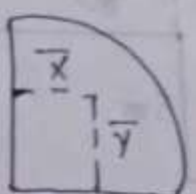
Semicircle

$$\text{Area} = \frac{\pi r^2}{2}$$

$$\bar{y} = \frac{4r}{3\pi}\ (\text{from bo}$$

$$\bar{x} = R$$

Quater circle

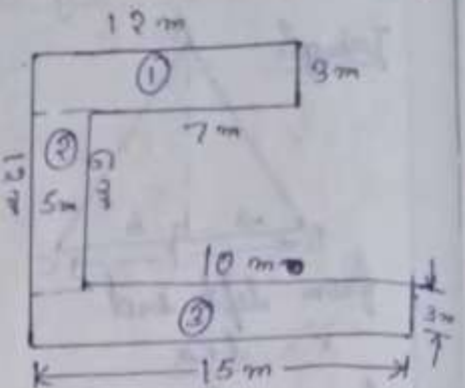


$$\bar{x} = \frac{4r}{3\pi}$$

$$\bar{y} = \frac{4r}{3\pi}$$

Area = $\frac{\pi r^2}{\ldots}$

① Find the centroid of the lamina as shown in fig



Ans: In fig ①

$A_1 = 12 \times 3 = 36 \, m^2$

$x_1 = \frac{12}{2} = 6 m$

$y_1 = 3 + 6 + \frac{3}{2} = 10.5 m$

In fig ②

$A_2 = 6 \times 5 = 30 m$

$x_2 = \frac{5}{2} = 2.5 m$

$y_2 = 3 + \frac{6}{2} = 3 + 3 = 6 m$

In fig ③

$A_3 = 15 \times 3 = 45 m$

$x_3 = \frac{15}{2} = 7.5 m$

$y_3 = \frac{3}{2} = 1.5 m$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{36 \times 6 + 30 \times 2.5 + 45 \times 7.5}{36 + 30 + 45}$$

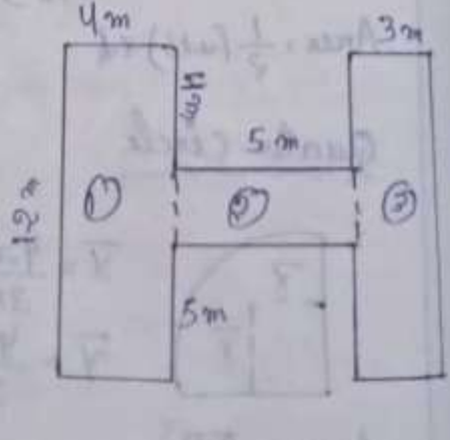
$$= 5.66 m$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{36 \times 10.5 + 30 \times 6 + 45 \times 1.5}{36 + 30 + 45}$$

$$= 5.635 m$$

② Find the centroid of the lamina as shown in fig.



Ans In fig ①

$A_1 = 12 \times 4 = 48 m$

$x_1 = \frac{4}{2} = 2 m$

$y_1 = \frac{12}{2} = 6 m$

In fig ②

$A_2 = 5 \times 3 = 15 m$

$X_2 = 4 + \frac{5}{2} = 4 + 2.5 = 6.5\ m$

$Y_2 = 5 + \frac{3}{2} = 6.5\ m$

In fig ③

$A_3 = 12 \times 3 = 36\ m$

$X_3 = 4 + 5 + \frac{3}{2} = 10.5\ m$

$Y_3 = \frac{12}{2} = 6\ m$

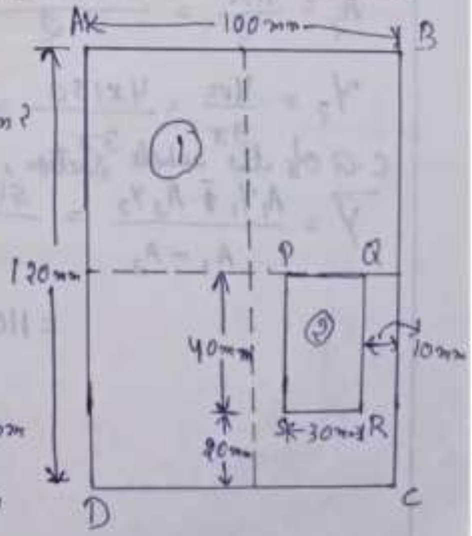
$$\overline{X} = \frac{A_1X_1 + A_2X_2 + A_3X_3}{A_1 + A_2 + A_3}$$

$$= \frac{48 \times 2 + 15 \times 6.5 + 36 \times 10.5}{48 + 15 + 36}$$

$$= 5.77\ m$$

$$\overline{Y} = \frac{48 \times 6 + 15 \times 6.5 + 36 \times 6}{48 + 15 + 36} = 6.07\ m$$

③ In a rectangular lamina 100mm x 120mm a rectangular Opening PQRS 30mm x 40mm is made as shown in fig.



$A_1 = 100 \times 120 = 12000\ mm^2$

$X_1 = \frac{100}{2} = 50\ mm$

$Y_1 = \frac{120}{2} = 60\ mm$

Deducted area

$A_2 = 40 \times 30 = 1200\ mm^2$

$X_2 = 50 + 10 + \frac{30}{2} = 75\ mm$

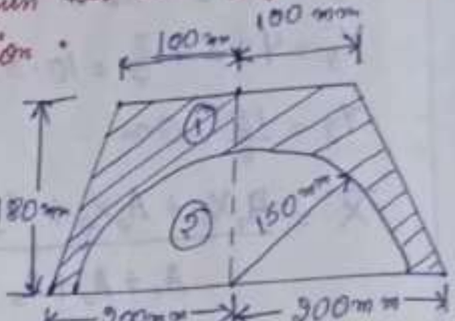
$Y_2 = 20 + \frac{40}{2} = 40\ mm$

$$\overline{X} = \frac{12000 \times 50 - 1200 \times 75}{12000 - 1200} = 47.22\ mm$$

(5)

$$\bar{y} = \frac{12000 \times 60 - 1200 \times 40}{12000 - 1200} = 62.22 \text{ mm}$$

(4) A semi circle of 150 mm radius is cut out from a trapezium as shown as the figure. Find the C.G. of the section.



Ans: In fig (1)

$$A_1 = \frac{1}{2}(a+b)\,h$$

$$= \frac{1}{2}(200+400) \times 180$$

$$= 54000 \text{ mm}^2$$

$$y_1 = \frac{h}{3}\left(\frac{b+2a}{b+a}\right)$$

$$= \frac{180}{3}\left(\frac{400+2\times 200}{400+200}\right) = 80 \text{ mm}$$

In fig (2)

$$A_2 = \frac{\pi r^2}{2} = \frac{\pi \times (150)^2}{2} = 35343 \text{ mm}^2$$

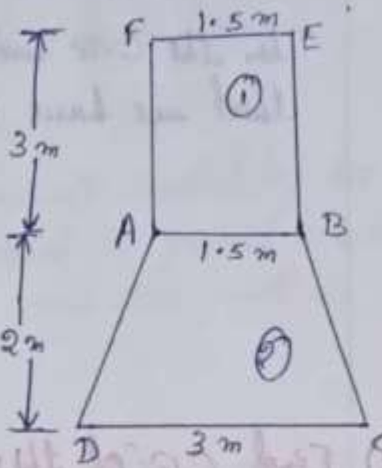
$$y_2 = \frac{4r}{3\pi} = \frac{4\times 150}{3\pi} = 63.66 \text{ mm}$$

C.G of the whole section,

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{54000 \times 80 - 35343 \times 63.66}{54000 - 35343}$$

$$= 110.95 \text{ mm}$$

(5) Find C.G of this section ?



In fig (1)

$A_1 = 1.5 \times 3 = 4.5\ m^2$

$y_1 = 2 + \frac{3}{2} = 3.5\ m$

In fig (2)

$A_2 = \frac{1}{2}(a+b)h$

$= \frac{1}{2} \times (1.5 + 3) \times 2$

$= 4.5\ m^2$

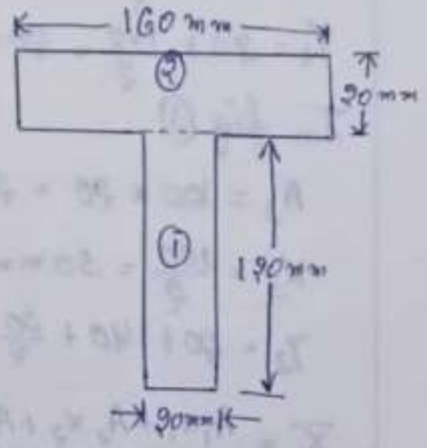
$y_2 = \frac{h}{3}\left(\frac{b+2a}{b+a}\right)$

$= \frac{2}{3}\left(\frac{3+2\times1.5}{3+1.5}\right) = 0.89\ m$

$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$= \frac{4.5 \times 3.5 + 4.5 \times 0.89}{4.5 + 4.5} = 2.19\ m$

(6) Find centroid of T section of flange 160×20 mm and web 120 mm × 20 mm.



Ans: In fig (1)

$A_1 = 120 \times 20 = 2400\ mm^2$

$X_1 = 70 + \frac{20}{2} = 70 + 10 = 80\ mm$

$Y_1 = \frac{120}{2} = 60\ mm$

In fig (2)

$A_1 = 160 \times 20 = 3200\ mm^2$

$X_1 = \frac{160}{2} = 80\ mm$

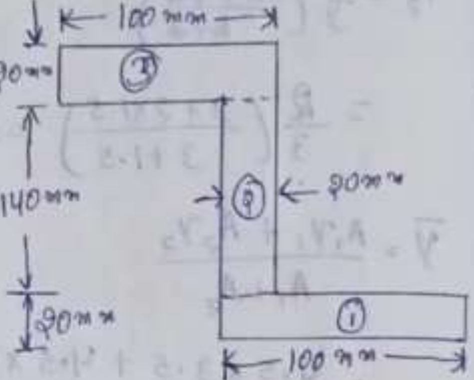
$Y_1 = 120 + \frac{20}{2} = 120 + 10 = 130\ mm$

As the section Simetrical about Y-Y axis. Then the C.G will lie on that axis after that we have to find $\overline{Y} = \frac{A_1 Y_1 + A_2 Y_2}{A_1 + A_2}$

$$= \frac{2400 \times 60 + 3200 \times 130}{2400 + 3200}$$

$$= 100 \text{ mm}$$

⑦ Find C.G of this section ?



Ans: In fig ①

$A_1 = 100 \times 20 = 2000 \text{ mm}^2$

$X_1 = 180 + \frac{100}{2}$

$= 130 \text{ mm}$

$Y_1 = \frac{20}{2} = 10 \text{ mm}$

In fig ②

$A_2 = 140 \times 20 = 2800 \text{ mm}^2$

$X_2 = 80 + \frac{20}{2} = 90 \text{ mm}$

$Y_2 = 20 + \frac{140}{2} = 20 + 70 = 90 \text{ mm}$

In fig ③

$A_3 = 100 \times 20 = 2000 \text{ mm}^2$

$X_3 = \frac{100}{2} = 50 \text{ mm}$

$Y_3 = 20 + 140 + \frac{20}{2} = 170 \text{ mm}$

$$\overline{X} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3} = \frac{2000 \times 130 + 2800 \times 90 + 2000 \times 50}{2000 + 2800 + 2000}$$

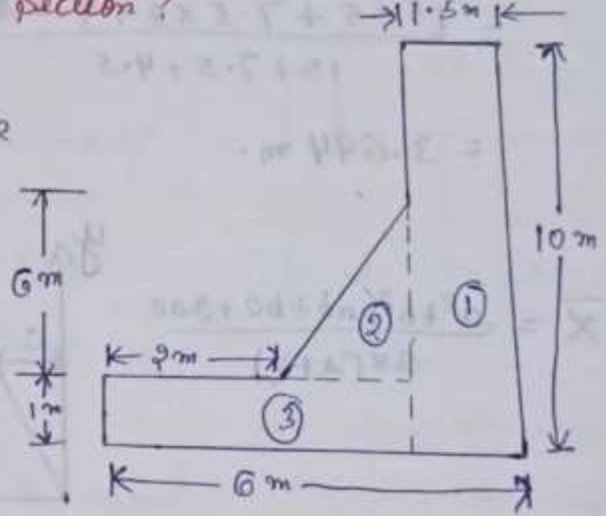
$$= 90 \text{ mm}$$

$$\bar{Y} = \frac{A_1Y_1 + A_2Y_2 + A_3Y_3}{A_1 + A_2 + A_3} = \frac{2000\times10 + 2800\times90 + 2000\times170}{2000 + 2800 + 2000}$$

$$= 90\,mm$$

* From area Point of view moment created by top portion about its C.G is equal to moment created by bottom area about C.G. and this will be Simetrycal about both axis.

⑧ Find C.G of this Section ?



In fig ①

$A_1 = 10 \times 1.5 = 15\,m^2$

$x_1 = \frac{1.5}{2} = 0.75\,m$

$Y_1 = \frac{10}{2} = 5m.$

In fig ②

~~$A_2 = \frac{1}{2} \times 4.5 \times 6$~~

$A_2 = \frac{1}{2} \times 2.5 \times 6$

$= 7.5\,m^2$

$x_1 = 1.5 + \frac{2.5}{3} = 2.33\,m.$

$Y_2 = \frac{h}{3} = \frac{6}{3} = 3\,m.$

In fig ③

$A_3 = 4.5 \times 1 = 4.5\,m^2$

$x_3 = \frac{4.5}{2} = 2.25 + 1.5 = 3.75\,m$

$Y_3 = \frac{1}{2} = 0.5\,m.$

$$\overline{X} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

$$= \frac{15\times 0.75 + 7.5\times 2.33 + 4.5\times 3.75}{15 + 7.5 + 4.5}$$

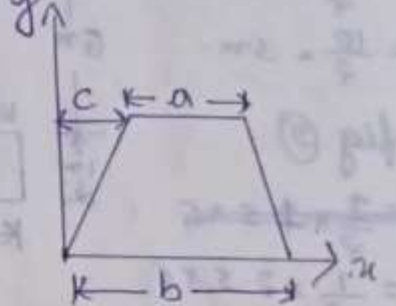
$$= 1.69 \text{ m}$$

$$\overline{Y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$$= \frac{15\times 5 + 7.5\times 3 + 4.5\times 0.5}{15 + 7.5 + 4.5}$$

$$= 3.694 \text{ m.}$$

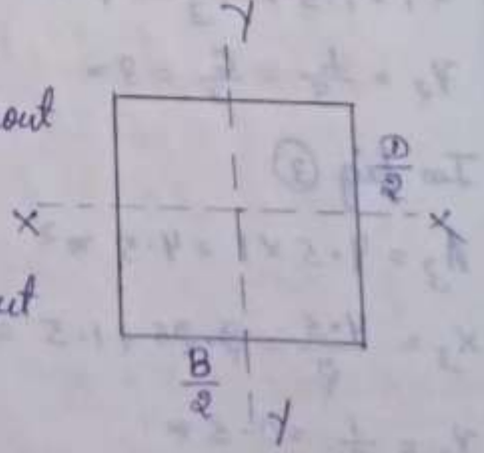
$$\overline{X} = \frac{a^2 + b^2 + ab + bc + 2ac}{3\times(a+b)}$$



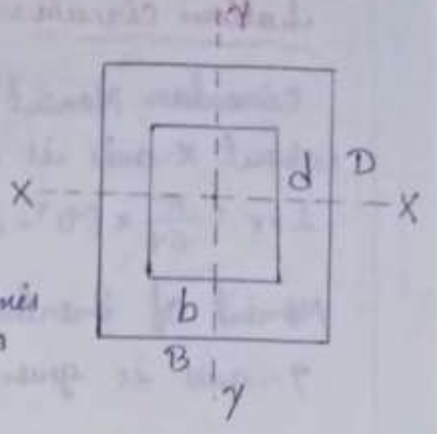
1) Reactangular Lamina :-

Moment of Inertia about x-axis $I_{xx} = \frac{bd^3}{12}$

Moment of Inertia about y-axis $I_{yy} = \frac{db^3}{12}$



2) Rectangular lamina with a centrally situated hollow rectangular :-

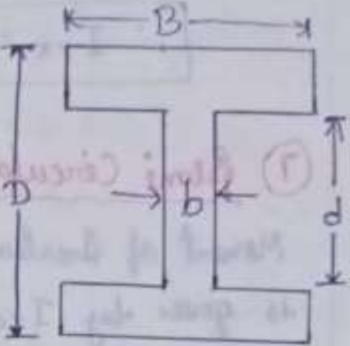


Moment of Inertia about x-axis is given by

$$I_{xx} = \frac{BD^3 - bd^3}{12}$$

Moment of Inertia about Y-axis is given by $I_{YY} = \frac{DB^3 - db^3}{12}$

3) 'I' Section :-

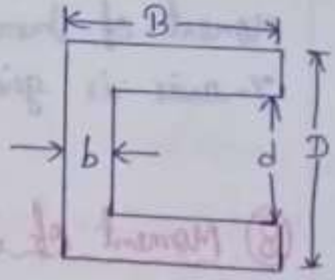


Moment of Inertia about x-axis is given by

$$I_{xx} = \frac{BD^3}{12} - \frac{(B-b)d^3}{12}$$

$$= \frac{1}{12}\left\{BD^3 - (B-b)d^3\right\}$$

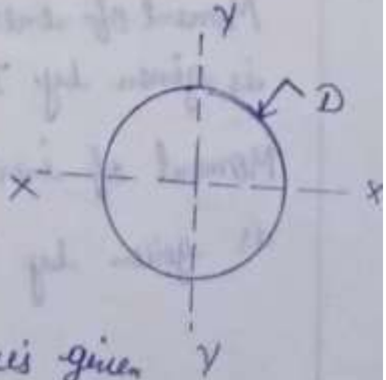
4) 'C' Section :-



Moment of Inertia about x-axis is given by

$$I_{xx} = \frac{BD^3}{12} - \frac{(B-b)d^3}{12}$$

5) Circular Lamina :-

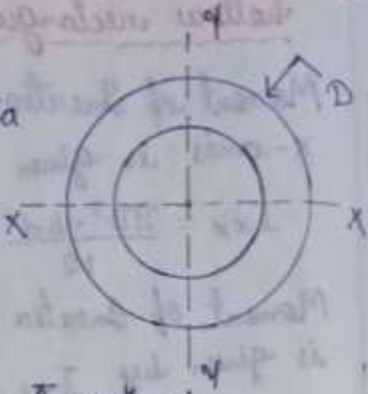


Moment of Inertia about x-axis is given by

$$I_{xx} = \frac{\pi \times D^4}{64}$$

Moment of Inertia about Y-axis given by $I_{YY} = \frac{\pi \times D^4}{64}$

⑥ Circular lamina with a centerally situated hollow circular -:



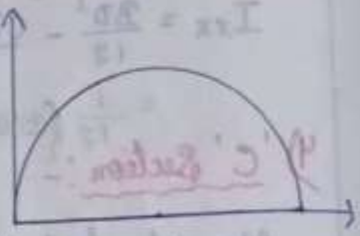
~~Circular~~ Moment of Inertia about X-axis is given by

$I_{xx} = \frac{\pi}{64} \times (D^4 - d^4)$

Moment of Inertia about Y-axis is given by $I_{YY} = \frac{\pi}{4} \times (D^4 - d^4)$

$$\boxed{\therefore I_{xx} = I_{yy} = \frac{\pi}{4} \times (D^4 - d^4)}$$

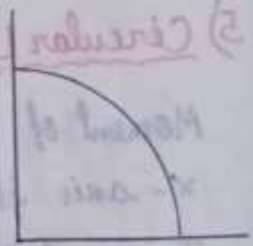
⑦ Semi Circular lamina about its Base or diameter:-



Moment of Inertia about x-axis is given by $I_{xx} = \frac{\pi d^4}{4}, 0.11 r^4$

Moment of Inertia about Y-axis is given by $I_{yy} = \frac{\pi d^4}{128}$ or $\frac{\pi r^4}{8}$

⑧ Moment of inertia qualar circle :-

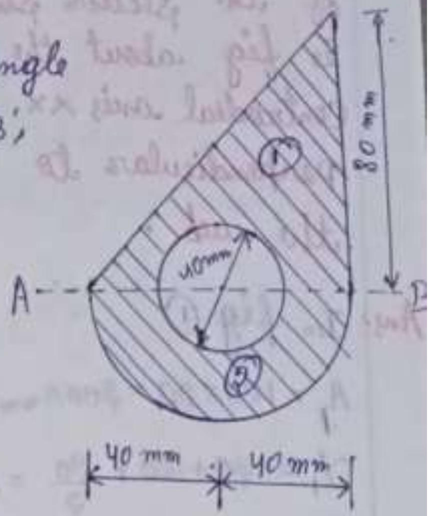


Moment of inertia about x-axis is given by $I_{xx} = \frac{1}{2} \times 0.11 \times R^4$

Moment of inertia about Y-axis is given by $I_{YY} = \frac{1}{2} \times \frac{\pi r^4}{8}$

I.M.P

(1) Find the moment of inertia of the area Shaded in fig about the axis AB.



Ans: Moment of inertia of right angle triangle about its base 'AB',

is $\frac{bh^3}{12}$

$= \frac{80 \times (80)^3}{12}$

$= 3413333.33\ mm^4$

Moment of Inertia of the Semi circle about its diameter 'AB',

$I_{xx} = \frac{\pi d^4}{128} = \frac{\pi \times (80)^4}{128} = 1005309.65\ mm^4$

Total moment of Inertia of the given (1) & (2) fig about the axis $AB = 3413333.33 + 1005309.65$

$= 4418642.98\ mm^4$

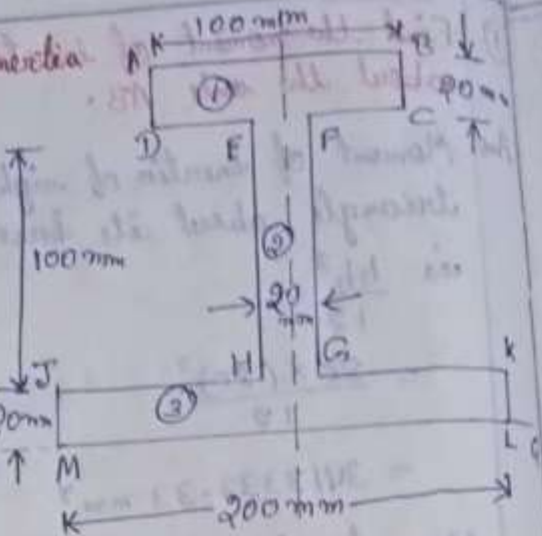
Now moment of Inertia of the circle of 40 mm dia about 'AB' $= \frac{\pi}{64} \times D^4 = \frac{\pi}{64} \times (40)^4 = 125663.71\ mm^4$

Hence the M.I of the Shaded portion

$= 4418642.98 - 125663.71$

$= 4292979.27\ mm^4$.

(2) Find the moment of inertia of the Section Show is fig about the Centrodial axis XX perpendicular to the web.



Ans: In fig (1)

$A_1 = 100 \times 20 = 2000\ mm^2$

$y_1 = 20 + 100 + \frac{20}{2} = 130\ mm$

$I_{G_1} = \frac{bd^3}{12} = \frac{100 \times (20)^3}{12} = 66666.67\ mm^4$

In fig (2)

$A_2 = 100 \times 20 = 2000\ mm^2$

$y_2 = 20 + \frac{100}{2} = 70\ mm$

$I_{G_2} = \frac{bd^3}{12} = \frac{20 \times (100)^3}{12} = 1666666.67\ mm^4$

In fig (3)

$A_3 = 200 \times 20 = 4000\ mm^2$

$y_3 = \frac{20}{2} = 10\ mm$

$I_{G_3} = \frac{bd^3}{12} = \frac{200 \times (20)^3}{12} = 133333.33\ mm^4$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = \frac{2000 \times 130 + 2000 \times 70 + 4000 \times 10}{2000 + 2000 + 400}$$

$= 55\ mm.$

Now, distance of x-x axis from PQ = 55 m ($\bar{Y}$)

~~M.I about axis PQ~~ $= 6666667 + 133333.33$
$+ 1666666.67$
$= 1866666.67 \text{ mm}^4$

$$\Sigma ay^2 = a_1 y_1^2 + a_2 y_2^2 + a_3 y_3^2$$
$$= 44000000 \text{ mm}^4$$

$$\Sigma M.I = 1866667 \text{ mm}^4$$

M.I about the axis PQ $= \Sigma M.I + \Sigma ay^2$
$= 1866667 + 44000000$
$= 45866667 \text{ mm}^4$

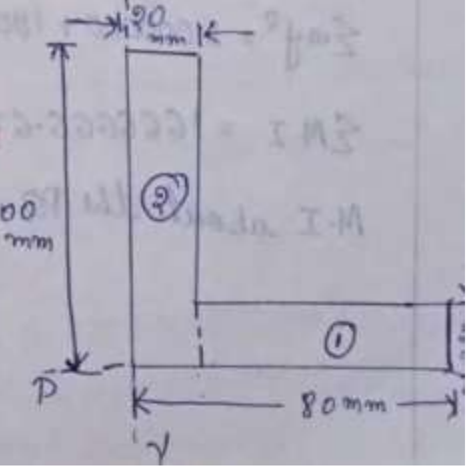
We have $I_{PQ} = I_{xx} + \Sigma a\bar{y}^2$

$\Rightarrow 45866667 = I_{xx} + 8000 \times (55)^2$

$\Rightarrow I_{xx} = 45866667 - 8000 \times (55)^2$

$= 21666667 \text{ mm}^4$.

③ Find the M.I of the Centrodial axis xx and YY of the section shown in the fig.



Ans: In fig ①

$A_1 = 60 \times 20 = 1200 \text{ mm}^2$

$x_1 = 20 + \frac{60}{2} = 50 \text{ mm}$

$Y_1 = \frac{20}{2} = 10 \text{ mm}$

In fig ②

$A_2 = 100 \times 20 = 2000 \text{ mm}^2$

$X_2 = \frac{20}{2} = 10 \text{ mm}$

$Y_2 = \frac{100}{2} = 50 \text{ mm}$

$$\overline{X} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{1200 \times 60 + 2000 \times 10}{1200 + 2000} = \cancel{28.75 \text{ mm}} \; 25 \text{ mm}$$

$$\overline{Y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{1200 \times 10 + 2000 \times 50}{1200 + 2000} = 35 \text{ mm}$$

M.I of fig (1) about its centrodial axis I_{G_1}

$$\frac{bd^3}{12} = \frac{60 \times (20)^3}{12} = 40000 \text{ mm}^4$$

M.I of fig (2) about its centrodial axis I_{G_2}

$$\frac{bd^3}{12} = \frac{20 \times (100)^3}{12} = 1666666.67 \text{ mm}^4$$

In fig (i) $ay^2 = 1200 \times (10)^2 = 120000 \text{ mm}^4$

In fig (ii) $Ay^2 = 2000 \times (50)^2 = 5000000 \text{ mm}^4$

$\Sigma ay^2 = 5000000 + 120000 = 5120000 \text{ mm}^4$

$\Sigma M.I = 1666666.67 + 40000 = 1706666.67 \text{ mm}^4$

M.I about the PQ axis $= \Sigma M.I + \Sigma ay^2$

$= 1706666.67 + 5120000$

$= 6826666.67 \text{ mm}^4$

we have M.I about the axis (PQ) = $I_{xx} + A\bar{y}^2$

$\Rightarrow 6826666.67 = I_{xx} + 3200 \times (35)^2$

$\Rightarrow I_{xx} = 6826666.67 - 3200 \times (35)^2$

$\Rightarrow \boxed{I_{xx} = 2906666.67 \text{ mm}^4}$

we known that moment of inertia of rectangle (1) about an axis through its center of gravity and parallel to Y-Y axis.

$$I_{G_1} = \frac{db^3}{12} = \frac{20 \times (60)^3}{12} = 360000 \text{ mm}^4$$

M.I about rectangular (2)

$$I_{G_2} = \frac{db^3}{12} = \frac{100 \times (20)^3}{12} = 66666.67 \text{ mm}^4$$

In fig (1) $ax^2 = 1200 \times (50)^2 = 3000000 \text{ mm}^4$

In fig (2) $ax^2 = 2000 \times (10)^2 = 200000 \text{ mm}^4$

$\Sigma ay^2 = 3000000 + 200000 = 3200000 \text{ mm}^4$

$\Sigma M.I = 360000 + 66666.67 = 426666.67$

M.I about the XY axis $= \Sigma M.I + \Sigma ay^2$

$= 426666.67 + 3200000$

$= 3626666.67 \text{ mm}^4$

we have M.I about the axis ($\cancel{\ }$ x Y)

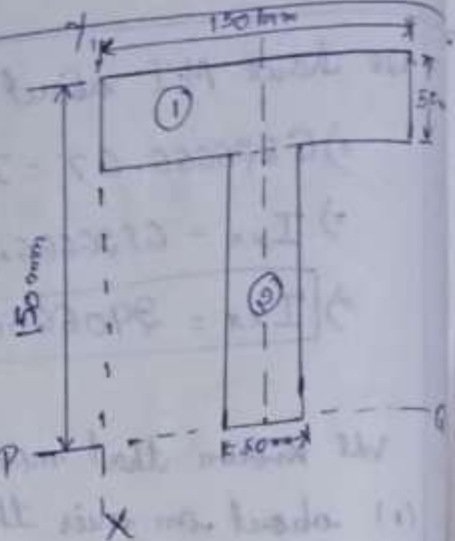
$= I_{yy} + \Sigma a\bar{y}^2$

$\Rightarrow 3626666.67 = I_{xy} + 3200 \times \cancel{(28.75)^2} (25)^2$

$\Rightarrow I_{xy} = 3626666.67 - 3200 \times \cancel{(28.75)^2} (25)^2$

$\Rightarrow I_{xy} = 1626666.67 \text{ mm}^4$

4) Find the M.I of the centroid axis XX and YY of the section shown in the fig.



Ans: In fig ①

$A_1 = 150 \times 50 = 7500\ mm^2$

$x_1 = \frac{150}{2} = 75\ mm$

$y_1 = 100 + \frac{50}{2} = 125\ mm$

In fig ②

$A_2 = 100 \times 50 = 5000\ mm^2$

$x_2 = 50 + \frac{50}{2} = 75\ mm$

$y_2 = \frac{100}{2} = 50\ mm$

$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{7500 \times 75 + 5000 \times 75}{7500 + 7500}$

$= 75\ mm$

$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{7500 \times 125 + 5000 \times 50}{7500 + 7500} = 95\ mm$

In Ixx axis

M.I of fig (i) about its centroid axis I_{G_1}

$= \frac{bd^3}{12} = \frac{150 \times (50)^3}{12} = 1562500\ mm^4$

M.I of fig (ii) about its centroid axis I_{G_2}

$= \frac{bd^3}{12} = \frac{50 \times (100)^3}{12} = 4166666.67\ mm^4$

$\Sigma M.I = 1562500 + 4166666.67 = 5729166.67\ mm^4$

$\Sigma ay^2 = a_1 y_1^2 + a_2 y_2^2$

$= 129687500 \, mm^4$

M.I. about PQ axis $= \Sigma M.I + \Sigma ay^2$

$= 5729166.67 + 129687500$

$= 135416666.7 \, mm^4$

$I_{xx} = I_{PQ} - \Sigma a\bar{y}^2$

$= 135416666.7 - 12500 \times (95)^2$

$= 22604166.7 \, mm^4$

<u>In I_{YY} axis</u>

M.I of fig (i) about its centroid axis I_{G_1}

$\frac{db^3}{12} = \frac{50 \times (150)^3}{12} = 14062500 \, mm^4$

M.I of fig (ii) about its Centroid axis I_{G_2}

$\frac{db^3}{12} = \frac{100 \times (50)^3}{12} = 1041666.67 \, mm^4$

$\Sigma M.I = 15104166.67 \, mm^4$

$\Sigma ax^2 = a_1 x_1^2 + a_2 x_2^2 = 70312500 \, mm^4$

M.I about XY axis $= \Sigma M.I + \Sigma ax^2$

$= 15104166.67 + 70312500$

$= 85416666.67 \, mm^4$

$I_{YY} = I_{XY} - \Sigma a\bar{x}^2$

$= 85416666.67 - 12500 \times (75)^2$

$= 15104166.67 \, mm^4$

Chapter 3

Stresses inBeams .

* Stresses in beam due to bending: 24.12.21

→ A member subjected to bending moment and shear forces undergoes certain deformation, so that the material of the member will offer resistance or stresses against these deformation.

→ A bending moment bends a member. So stresses induced by bending moment are called bending stresses.

→ Similarly stresses induced due to shear forces are called shear stresses.

* Pure bending: (2 marks)

When any portion of a beam is absolutely free from shear force but subjected to bending moment, in that case the bending is called pure bending (or) simple bending.

* Moment of Resistance:

It is defined as "The maximum bending moment, a section can resist."

(OR)

It is defined as "The couple produced by the internal forces in a beam subjected to bending under the maximum permissible stress."

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* Assumption in the theory of simple bending or pure bending: (6 marks)

Imp.

1. The beam is subjected to pure bending and is therefore free from shear force.
2. The material of the beam is homogeneous and isotropic.
3. The value of the Young's Modulus is the same for the beam material in tension as well as in compression.
4. A transverse section of the beam which is a plane before bending will remain a plane after bending.
5. The resultant pull or thrust on a transverse section of the beam is zero.
6. The transverse section of the beam is symmetrical about an axis passing through the centroid of the section and parallel to the plane of bending.

7. The radius of curvature of the deflected beam is very large compared with the dimensions of the cross-section of the beam.

[neutral axis → divides the tension and compression]

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* Simple Bending Equation:

$$\boxed{\frac{M}{I} = \frac{f}{y} = \frac{E}{R}}$$

where, M → Bending Moment

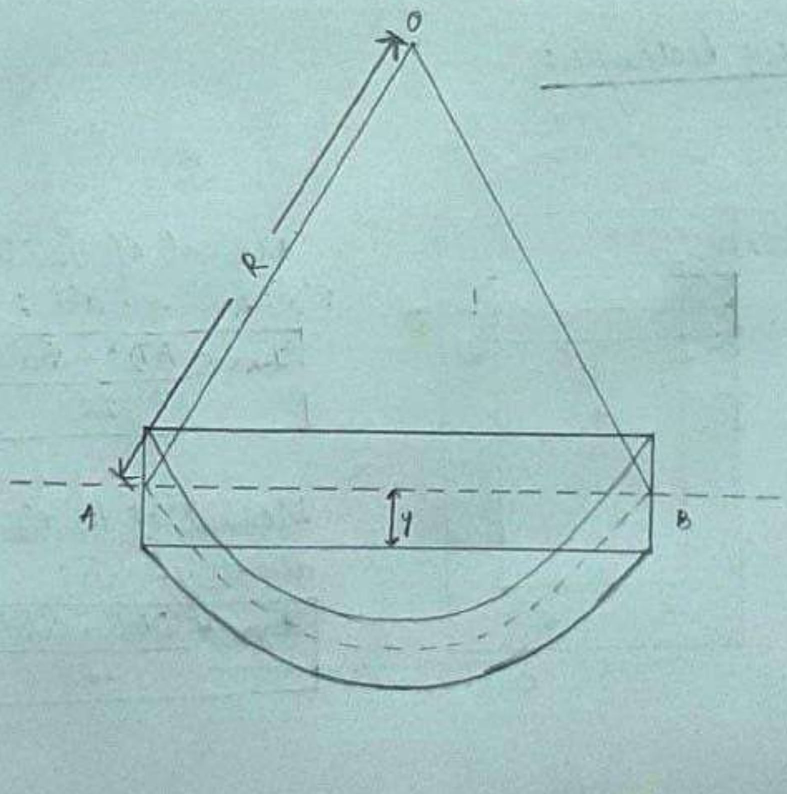
F → Stress on the beam

I → Moment of Inertia about the neutral axis

y → distance of neutral axis

E → Young's Modulus of Elasticity

R → Radius of Bending Arc



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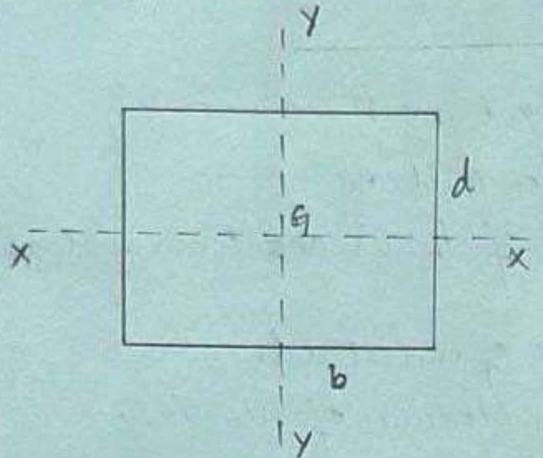
* Some formula Related to moment of Inertia :

① Rectangle :



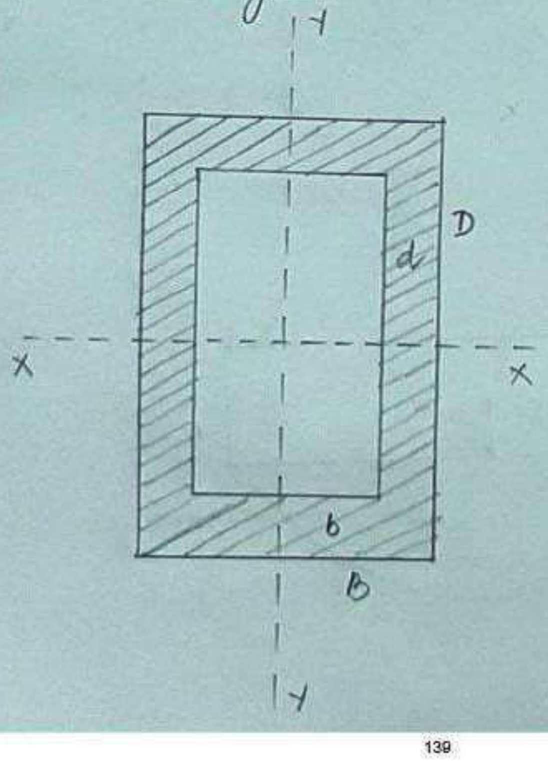
Moment of Inertia about x-axis:

$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

Moment of Inertia about y-axis:

$$\boxed{I_{YY} = \frac{db^3}{12}}$$

② Hollow Rectangle :



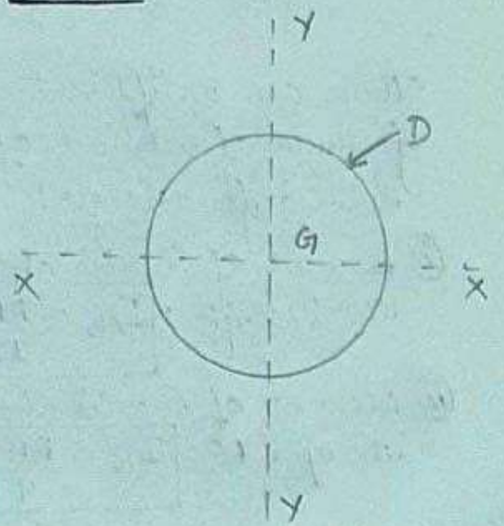
Moment of Inertia about x-axis:

$$\boxed{I_{xx} = \frac{BD^3 - bd^3}{12}}$$

Moment of Inertia about y-axis:

$$\boxed{I_{YY} = \frac{DB^3 - db^3}{12}}$$

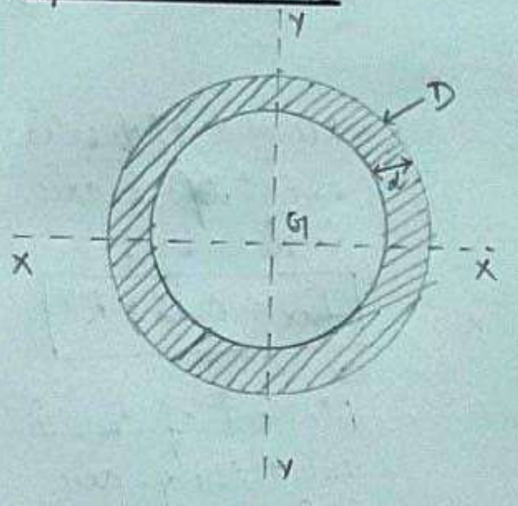
③ Circle:



Moment of Inertia about the x-axis and y-axis will be same, which is given by:

$$\boxed{I_{xx} = I_{yy} = \frac{\pi}{64} \times D^4}$$

④ Hollow Circle:



Moment of Inertia about the x-axis and y-axis will be same, which is given by:

$$\boxed{I_{xx} = I_{yy} = \frac{\pi}{64}(D^4 - d^4)}$$

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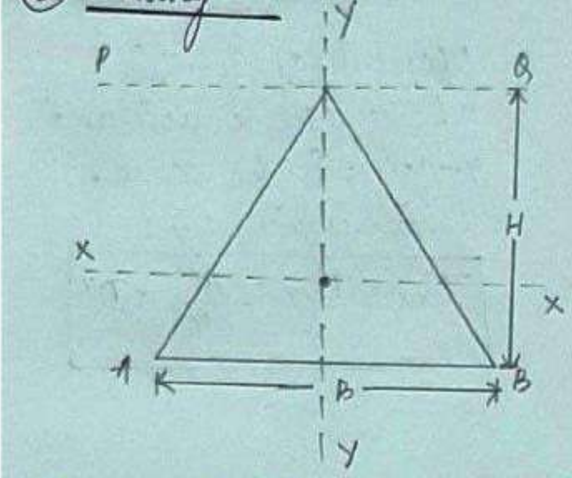
⑤ Triangle:



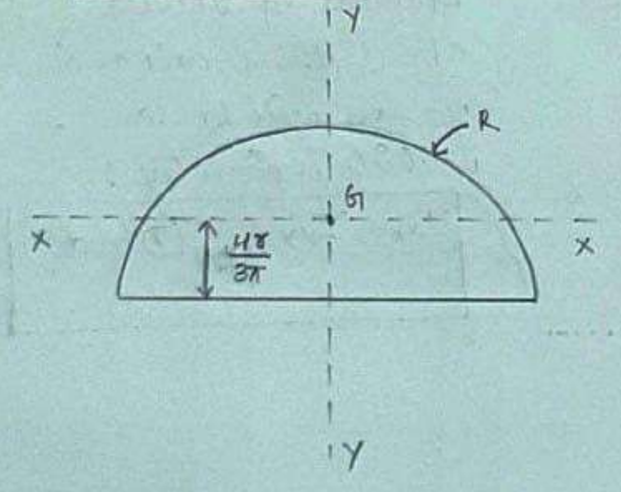
There are 3 different formula of moment of inertia for triangle:

(i) Moment of inertia about the base AB : $\boxed{I_{AB} = \frac{BH^3}{12}}$

(ii) Moment of inertia about the apex PQ : $\boxed{I_{PQ} = \frac{BH^3}{4}}$

(iii) Moment of Inertia about its centroid i.e : $\boxed{I_{xx} = \frac{BH^3}{36}}$

⑥ Semi-Circle:



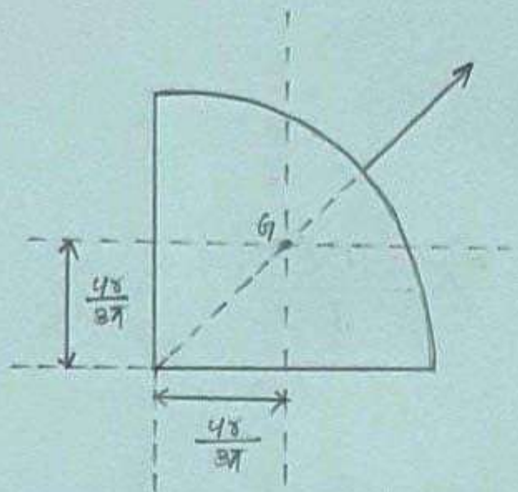
Moment of Inertia about the x-axis is given by :

$$\boxed{I_{xx} = 0.11 \times R^4}$$

Moment of Inertia about the y-axis is given by :

$$\boxed{I_{yy} = \frac{\pi}{8} \times R^4 \Rightarrow I_{yy} = 0.393 \times R^4}$$

(7) Quarter Circle:

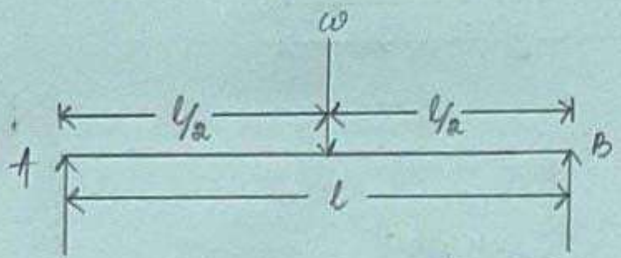


Moment of Inertia about the y-axis same, which is given by:

$$\boxed{I_{xx} = I_{yy} = \frac{0.11}{2} \times R^4}$$

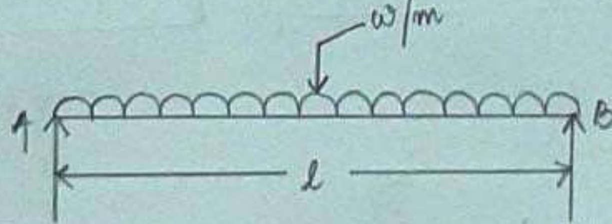
* Maximum Bending Moment:

(1)



$$\boxed{BM = \frac{wl}{4}}$$

(2)



$$\boxed{BM = \frac{wl^2}{8}} \quad \text{(or)} \quad \boxed{BM = \frac{Wl}{8}} \quad [\because W = wl]$$

where, W is the total load.

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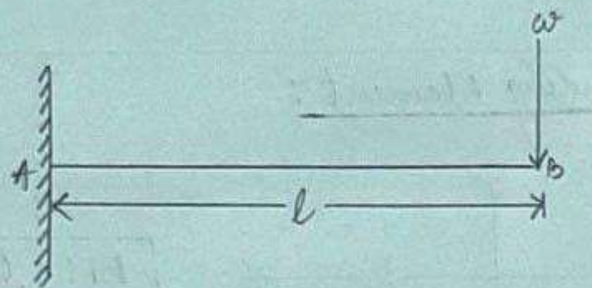
$$\Rightarrow \left(\frac{wl}{2} \times \frac{l}{2}\right) - \left(\frac{wl}{2} \times \frac{l}{4}\right)$$

$$\Rightarrow \frac{wl^2}{4} - \frac{wl^2}{8}$$

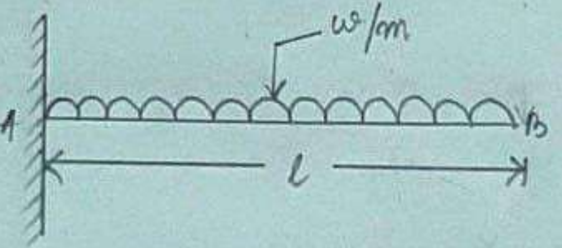
$$\Rightarrow \frac{2wl^2 - wl^2}{8} \Rightarrow \frac{wl^2}{8}.$$

③



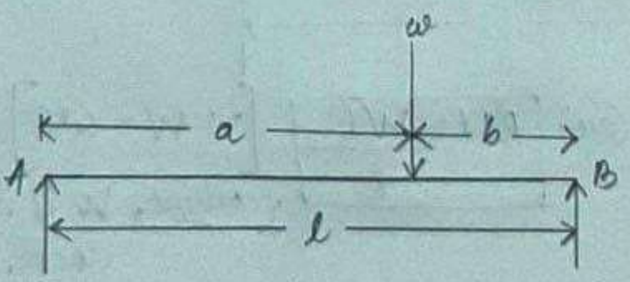
$$\boxed{BM = wl}$$

④



$$\boxed{BM = \frac{wl^2}{2}}$$

⑤



$$\boxed{BM = \frac{Wab}{l}}$$

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* Neutral Axis:

28.12.21

→ The imaginary axis in a beam which divides a beam into two zones, tension and compression.

→ This is also called <u>zero stress line</u> (or) <u>zero stress axis</u>.

* Section Modulus:

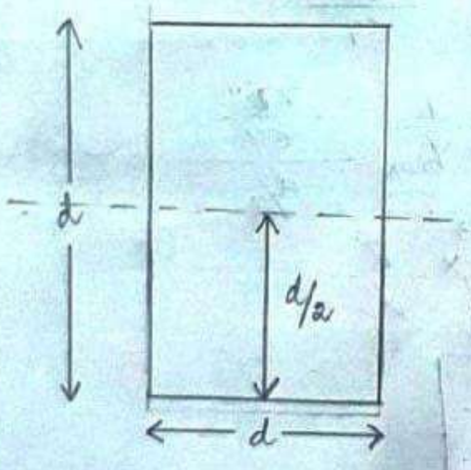
→ It is defined as "The ratio of moment of inertia about the neutral axis to the distance of the most distant point of the section from the neutral axis."

→ It is denoted as 'Z'.

→ Mathematically, $\boxed{Z = \dfrac{I}{Y_{max}}}$

* Section Modulus for Various Shape of Beam:

① Rectangular Section:



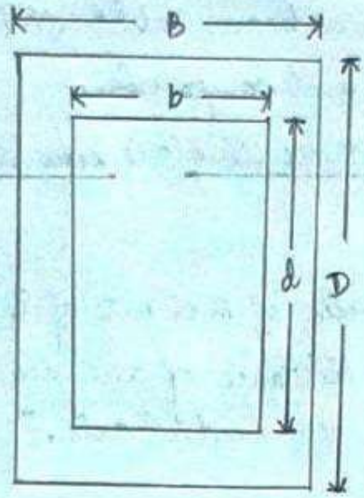
$$Y_{max} = \frac{d}{2}$$

$$I = \frac{bd^3}{12}$$

$$Z = \frac{I}{Y_{max}} = \frac{\frac{bd^3}{12}}{\frac{d}{2}}$$

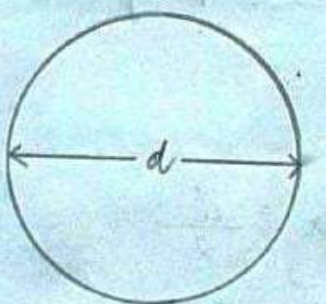
$$= \frac{bd^{\cancel{3}\,2}}{\cancel{12}\,6} \times \frac{\cancel{2}}{\cancel{d}} \quad \boxed{Z = \frac{bd^2}{6}}$$

② Hollow Rectangular Section:



$$\boxed{Z = \frac{BD^3 - bd^3}{6D}}$$

③ Circular section:



$$Y_{max} = \frac{d}{2}$$

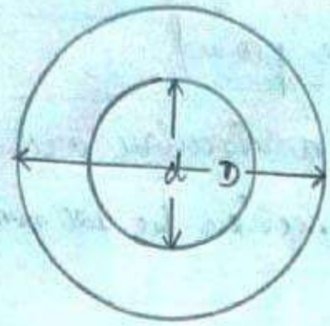
$$I = \frac{\pi d^4}{64}$$

$$Z = \frac{I}{Y_{max}} = \frac{\frac{\pi d^4}{64}}{d/2}$$

$$\Rightarrow Z = \frac{\pi d^{\cancel{4}\,3}}{\cancel{64}_{32}} \times \frac{\cancel{2}}{\cancel{d}}$$

$$\Rightarrow \boxed{Z = \frac{\pi d^3}{32}}$$

(4) Hollow Circular Section:



$$Y_{max} = \frac{d}{2}$$

$$I = \frac{\pi(D^4 - d^4)}{64}$$

$$Z = \frac{I}{Y_{max}} = \frac{\frac{\pi(D^4-d^4)}{64}}{\frac{d}{2}}$$

$$\Rightarrow Z = \frac{\pi(D^4-d^4)}{\cancel{64}\,32} \times \frac{\cancel{2}}{d}$$

$$\Rightarrow \boxed{Z = \frac{\pi(D^4-d^4)}{32d}}$$

---

* **Relationship Between Maximum Bending Moment (M), Stress (F) and Section Modulus (Z):**

$$\boxed{M = F \times Z}$$

Proof: In pure bending equation,

$$\Rightarrow \frac{M}{I} = \frac{F}{y}$$

$$\Rightarrow \frac{M}{F} = \frac{I}{y}$$

$$\Rightarrow \frac{M}{F} = Z \qquad \left[\because Z = \frac{I}{y}\right]$$

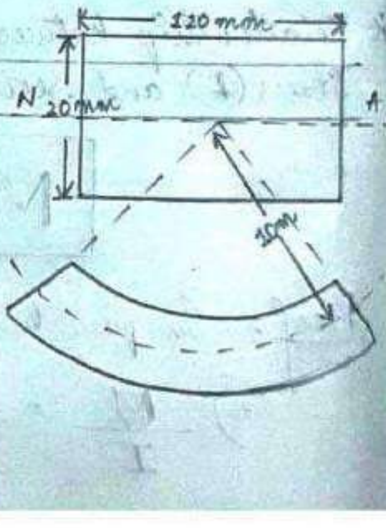
$$\Rightarrow \boxed{M = F \times Z}$$

hence proved

① A steel plate is bent into a circular arc of radius 10 m. If the plane section be 120 mm wide and 20 mm thick.
Find the maximum stress induced and the bending moment which can produce this stress.
Take $E = 2 \times 10^5 \ N/mm^2$.

given:) radius $(R) = 10 m$
$= 10 \times 10^3 \ mm$

breadth $(B) = 120 mm$
thickness $(D) = 20 mm$
$E = 2 \times 10^5 \ N/mm^2$.



$$I = \frac{bd^3}{12} = \frac{120 \times (20)^3}{12}$$

$$I = 80000 = 8 \times 10^4 \ mm^4$$

we know for pure bending,

$$\boxed{\frac{M}{I} = \frac{F}{y} = \frac{E}{R}}$$

$$\Rightarrow \frac{M}{I} = \frac{E}{R}$$

$$\Rightarrow M = \frac{E \times I}{R}$$

$$\Rightarrow M = \frac{2 \times 10^5 \times 8 \times 10^4}{10 \times 10^3}$$

$$\Rightarrow \boxed{M = 1.6 \times 10^6 \text{ N-mm}}$$

$$\therefore Y_{max} = \frac{d}{2} = \frac{20}{2} = 10 \text{mm}$$

$$\frac{F}{y} = \frac{E}{R}$$

$$\Rightarrow F = \frac{E \times y}{R}$$

$$\Rightarrow F = \frac{2 \times 10^5 \times 10}{10 \times 10^3}$$

$$\Rightarrow \boxed{F = 200 \text{ N/mm}^2}$$

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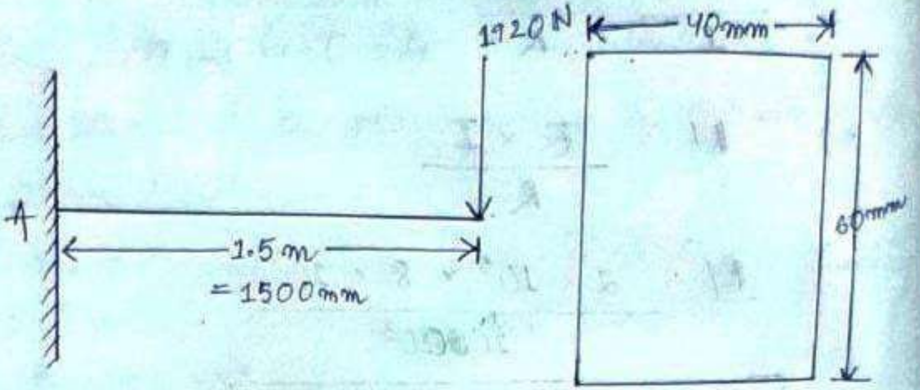
② A cast iron cantilever of length 1.5 m fails when a load of 1920 N is applied at the free end. Determine the stress at failure if the section of cantilever is 40 mm × 60 mm.



$BM = (1920 \times 1500)$
$= 2.88 \times 10^6$ N-mm

given) Beam size = 40 mm × 60 mm
point load = 1920 N
length of the beam = 1.5 m
Maximum BM = 2.88×10^6 N-mm
we know, $\boxed{M = fZ}$

$$Z = \frac{bd^2}{6} = \frac{40 \times (60)^2}{6}$$

$= 24000$ mm

$= 24 \times 10^3$ mm

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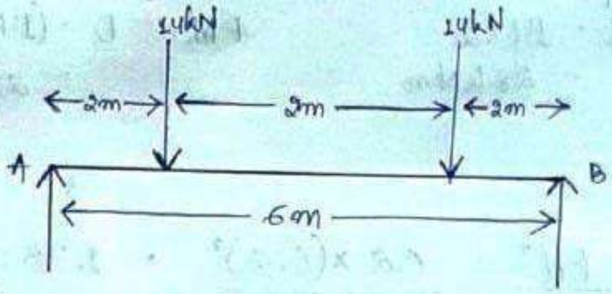
$$F = \frac{M}{Z} = \frac{2.88 \times 10^6}{24 \times 10^3}$$

$$\Rightarrow \boxed{F = 120 \frac{N}{mm^2}}$$

③ A wooden beam 200 mm × 200mm is simply supported on a span of 6m. When the beam is loaded with 14kN load at each $\frac{1}{3}$rd span point, it failed. Find the stress at failure.



*Reaction Calculation:

Taking moment about A = 0

All the clockwise moments = All the anticlockwise moments

$$\Rightarrow (14 \times 2) + (14 \times 4) = R_B \times 6$$

$$\Rightarrow R_B = \frac{(14 \times 2) + (14 \times 4)}{6}$$

$$\Rightarrow \boxed{R_B = 14 kN}$$

$\therefore$ Total load $= 14 + 14$

$\Rightarrow R_A + R_B = 28$

$\Rightarrow R_A = 28 - R_B$

$\Rightarrow R_A = 28 - 14$

$\Rightarrow \boxed{R_A = 14\ kN}$

(ii) <u>Bending Moment Calculation:</u>

BM at A $= 0$ BM at B $= 0$

BM at C $= 14 \times 2$
$= 28\ kNm$

BM at D $= (14 \times 4) - (14 \times 2)$
$= 28\ kNm$

$$Z = \frac{bd^2}{6} = \frac{0.2 \times (0.2)^2}{6} = 1.33 \times 10^{-3}\ m^3$$

$\therefore M = F \times Z$

$\Rightarrow F = \dfrac{M}{Z}$

$\Rightarrow F = \dfrac{28}{1.33 \times 10^{-3}} \dfrac{kN}{m^2}$

$\Rightarrow \boxed{F = 21052.63\ \dfrac{kN}{m^3}}$

(4) A steel beam of symmetrical section is subjected to a uniform bending moment which produces a maximum stress of $100\ N/mm^2$. If the beam is 250mm deep. Find the radius to which the longitudinal axis of the beam will be bend.

Take $E = 2 \times 10^5\ N/mm^2$.

Soln) given: $F = 100\ N/mm^2$.

$d = 250mm$

$E = 2 \times 10^5\ N/mm^2$.

$$Y = \frac{d}{2} = \frac{250}{2} = 125$$

$$\therefore \frac{F}{y} = \frac{E}{R}$$

$$\Rightarrow R = \frac{E \times y}{F}$$

$$\Rightarrow R = \frac{2 \times 10^5 \times 125}{100}$$

$$\Rightarrow R = 25 \times 10^4\ mm$$

$$\Rightarrow R = \frac{25 \times 10^4}{10^3}$$

$$\Rightarrow \boxed{R = 250\ m}$$

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(5) Find the maximum stress produced in a round steel bar 50mm in diameter and 9m long. Due to its own weight when it is simply supported at its two ends, steel weights at $77000 \, N/m^3$.

Solu:) given:) $d = 50 \, mm$

$l = 9m$

$= 9000 \, mm$

$V = A \times l$

$= \frac{\pi}{4}(50)^2 \times 9000$

$= 17.67 \times 10^6 \, mm^3$

specific weight $= 77000 \frac{N}{m^3}$

$\Rightarrow \frac{\text{weight}}{\text{volume}} = 77000 \frac{N}{m^3}$

$\Rightarrow \frac{\text{weight}}{17.67 \times 10^6 \, mm^3} = 77000 \frac{N}{m^3}$

$$\left[\because 77000 \frac{N}{m^3} = 77000 \frac{N}{(10^3)^3 mm^3} = 77000 \times 10^{-9} \frac{N}{mm^3} \right]$$

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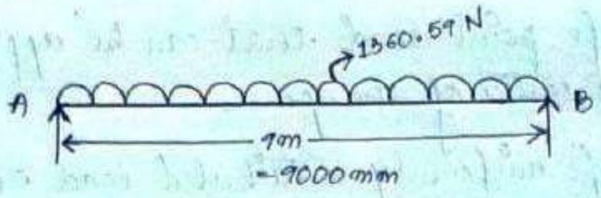
$$\Rightarrow \frac{\text{weight}}{17.67 \times 10^{6}\, mm^3} = 77000 \times 10^{-9} \frac{N}{mm^3}$$

$$\Rightarrow \text{weight} = 77000 \times 10^{-9} \times 17.67 \times 10^{6}$$

$$\Rightarrow \boxed{\text{weight} = 1360.59\ N}$$



we know,

$$BM = \frac{wl^2}{8}$$

$$= \frac{1360.59 \times (9 \times 10^{3})^{2}}{8}$$

$$\boxed{M = 1.37 \times 10^{10}\ N\text{-}mm}$$

$$\therefore M = f \times Z$$

$$\Rightarrow f = \frac{M}{Z}$$

$$\Rightarrow f = \frac{1.37 \times 10^{10}}{\frac{\pi}{32}(50)^3}$$

$$\Rightarrow \boxed{f = 1.11 \times 10^{6}\ N/mm^2}$$

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(6) A simply supported beam of span 10m is ~~85~~ 350 mm deep. The section of the beam is symmetrical. The moment of inertia of the section is $9.5 \times 10^7\ mm^4$. If the permissible bending stress is $120\ N/mm^2$, Find:

(i) the safe point load that can be applied at the centre of the span.

(ii) the safe uniformly distributed load that can be applied on the span.

(Neglect the dead load of the beam).

Soln) given: $l = 10m$

$= 10 \times 10^3\ mm$

$d = 350mm$

$I = 9.5 \times 10^7\ mm^4$

$f = 120\ N/mm^2$

Let M = maximum bending moment for the beam

$$\frac{M}{I} = \frac{f}{y}$$

$$\Rightarrow M = \frac{f \times I}{y}$$

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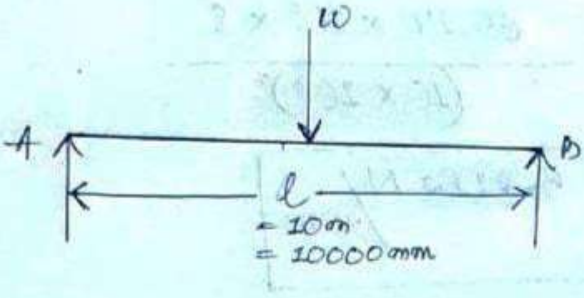
$$\Rightarrow M = \frac{120 \times 9.5 \times 10^7}{\frac{350}{2}}$$

$$\Rightarrow \boxed{M = 65.14 \times 10^6 \text{ N-mm}}$$

(i)



$$M = \frac{Wl}{4}$$

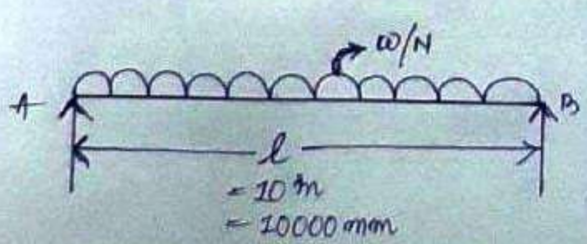
$$\Rightarrow 65.14 \times 10^6 = \frac{W \times (10 \times 10^3)}{4}$$

$$\Rightarrow W = \frac{65.14 \times 10^6 \times 4}{10 \times 10^3}$$

$$\Rightarrow \boxed{W = 26.056 \times 10^3 \text{ N}}$$

$$\Rightarrow \boxed{W = 26.056 \text{ kN}}$$

(ii)



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$$M = \frac{Wl^2}{8}$$

$$\Rightarrow 65.14 \times 10^6 = \frac{\omega \times (10 \times 10^3)^2}{8}$$

$$\Rightarrow \omega = \frac{65.14 \times 10^6 \times 8}{(10 \times 10^3)^2}$$

$$\Rightarrow \boxed{\omega = 5.2112 \, N/mm}$$

$$\Rightarrow \omega = \frac{5.2112}{10^3 \times 10^{-3}} \frac{kN}{m}$$

$$\Rightarrow \boxed{\omega = 5.2112 \, \frac{kN}{m}}$$

* <u>Derivation:</u>

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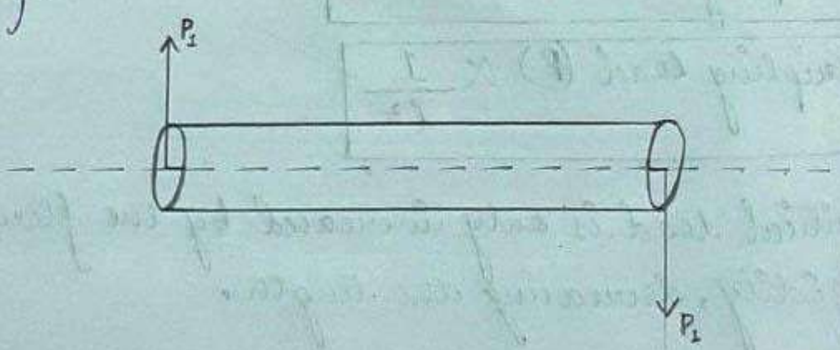
STRESSES IN SHAFTS DUE TO TORSION

* **Torsion:** The twisting of a body by the application of the forces tending to turn one end or part, about a longitudinal axis while the other and is held fixed or turned in the opposite direction is known as torsion.

* **Pure Torsion:** The shaft of circular section is said to be impure torsion when it is subjected to equal and opposite end couples whose axis coincide with the axis of the shaft.



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* Assumption of pure torsion:

1. The material of the shaft is uniform throughout.
2. The twist along the shaft is uniform.
3. The shaft is of uniform circular section throughout.
4. Cross-section of the shaft which are plain before twist, remain plain after twist.
5. All the radii which are straight before twist, remain straight after twist.

(1) Torsion of Solid shaft:

$$\boxed{T = \frac{\pi}{16} \tau \times D^3}$$

where, $T \rightarrow$ torque transmitted by shaft

$\tau \rightarrow$ shearing stress in N/mm^2.

$D \rightarrow$ diameter of shaft

(2) Torsion of Hollow shaft:

$$\boxed{T = \frac{\pi}{16} \tau \left[\frac{D^4 - d^4}{D}\right]}$$

where, $D \rightarrow$ external diameter

$d \rightarrow$ internal diameter

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(1) A hollow shaft of external and internal diameter 80mm and 50mm respectively is required to transmit torque from one end to the other end. What is the safe torque it can be transmit of the allowable shear stress in 45MPa.

given:)
$d = 50mm$
$D = 80mm$
$\tau = 45\ MPa$
$= 45\ N/mm^2$

$$T = \frac{\pi}{16}\tau\left[\frac{D^4 - d^4}{D}\right]$$

$$\Rightarrow T = \frac{\pi}{16} \times (45) \times \left[\frac{(80)^4 - (50)^4}{80}\right]$$

$$\Rightarrow \boxed{T = 3.83 \times 10^6\ N\text{-}mm}$$

$$\Rightarrow T = \frac{3.83 \times 10^6}{10^3 \times 10^3}\ kN\text{-}m$$

$$\Rightarrow \boxed{T = 3.83\ kN\text{-}m}$$

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(2) A steel shaft is to transmit a torque of 10 kNm. If the shearing stress is not to exceed 45 MPa, find the maximum diameter of the shaft.

given:) $T = 10\,kNm$
$= 10 \times 10^3 \times 10^3\ Nmm$
$= 10^7\ Nmm$

$\tau = 45\,MPa$
$= 45\,N/mm^2$

$\therefore\ T = \frac{\pi}{16}\tau \times D^3$

$\Rightarrow D^3 = \frac{T \times 16}{\pi \times \tau}$

$\Rightarrow D^3 = \frac{10^7 \times 16}{\pi \times 45}\ mm$

$\Rightarrow D^3 = 1.13 \times 10^6\ mm$

$\Rightarrow D = \sqrt{1.13 \times 10^6}\ mm$

$\Rightarrow \boxed{D = 104.15\ mm}$

$\Rightarrow D = \frac{104.15}{10^3}\ m$

$\Rightarrow \boxed{D = 0.10415\ m}$

* **Power Transmitted by a shaft:**

$$\boxed{P = \frac{2\pi NT}{60}}$$

→ Its unit: W, kW (watt, kilowatt)

where, P → power transmitted
N → no. of revolution per minute
T → average torque

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① A circular shaft of 60 mm diameter is rotating at 150 rpm. If the shear stress is not to exceed 50 MPa, find the power which can be transmitted by the shaft.

given:) $d = 60\,mm$

$N = 150\,rpm$

$\tau = 50\,MPa$

$= 50\,N/mm^2$.

$$T = \frac{\pi}{16}\tau d^3$$

$$\Rightarrow T = \frac{\pi}{16} \times (50) \times (60)^3$$

$$\Rightarrow \boxed{T = 2.12 \times 10^6\ N\text{-}mm}$$

$$\therefore P = \frac{2\pi NT}{60}$$

$$\Rightarrow P = \frac{2\pi \times 150 \times 2.12 \times 10^6}{60}$$

$$\Rightarrow \boxed{P = 33.3 \times 10^6\ N\text{-}m}$$

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$$\Rightarrow P = \frac{33.3 \times 10^6}{10^3 \times 10^3} \text{ kN-m}$$

$$\Rightarrow \boxed{P = 33.3 \text{ kN-m}}$$

31.12.21

(2) A hollow shaft of external and internal diameter as 100mm and 40mm respectively is transmitting power at 120rpm. find the power that the shaft can transmit, if the shearing stress is not to exceed 50 MPa.

given:) $D = 100\text{mm}$ $\quad N = 120\text{rpm}$

$d = 40\text{mm}$ $\quad \tau = 50\text{MPa}$

$= 50 \text{ N/mm}^2$

$$T = \frac{\pi}{16} \times \tau \left[\frac{D^4 - d^4}{D}\right]$$

$$\Rightarrow T = \frac{\pi}{16}(50) \times \left[\frac{(100)^4 - (40)^2}{100}\right]$$

$$\Rightarrow \boxed{T = 9.56 \times 10^6 \text{ N-mm}}$$

$$\therefore P = \frac{2\pi NT}{60}$$

$$\Rightarrow P = \frac{2\pi \times 120 \times 9.56 \times 10^6}{60}$$

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$\Rightarrow \boxed{P = 120.13 \times 10^6 \ W}$

③ A solid circular shaft of 100 mm diameter is transmitting 120 kW power at 150 rpm. Find the intensity of shear stress in the shaft.

given:) $d = 100$ mm

$N = 150$ rpm

$P = 120$ kW

$= 120 \times 10^3$ W

$$P = \frac{2\pi NT}{60}$$

$$\Rightarrow T = \frac{P \times 60}{2\pi N}$$

$$\Rightarrow T = \frac{120 \times 10^3 \times 60}{2\pi \times 150}$$

$$\Rightarrow \boxed{T = 7.63 \times 10^3 \ N\text{-mm}}$$

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$$\therefore T = \frac{\pi}{16} \tau d^3$$

$$\Rightarrow \tau = \frac{T \times 16}{\pi \times d^3}$$

$$\Rightarrow \tau = \frac{7.63 \times 10^3 \times 16}{\pi \times (100)^3}$$

$$\Rightarrow \boxed{\tau = 38.85 \times 10^{-3} \, N/mm^2}$$

or

$$\Rightarrow \boxed{\tau = 0.03 \, N/mm^2}$$

* Polar Moment of Inertia :

The moment of inertia of a plane area with respect to an axis perpendicular to the plane of the figure is called polar moment of inertia with respect to the point where the axis intersects the plane.

→ In a circular plane, the point is always the centre of the circle.

→ Polar moment of inertia is denoted as 'J'.

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$$\boxed{J = \frac{\pi}{32} D^4}$$ and $$\boxed{J = \frac{\pi}{32}(D^4 - d^4)}$$

(for solid shaft) (for hollow shaft)

* Polar Modulus (Z_p):

The ratio of polar moment of inertia of a shaft section to the maximum radius is called polar modulus.

mathematically,

$$\boxed{Z_p = \frac{\pi}{16} D^3}$$ and $$\boxed{Z_p = \frac{\pi}{16D}[D^4 - d^4]}$$

(for solid shaft) (for hollow shaft)

① A solid steel shaft has to transmit 100kN power at 160 rpm.
Taking allowable shear stress as 70 MPa, find the suitable diameter of the shaft.
The maximum torque transmitted in each revolution exceeds the torque by 20%.

given:) $P = 100\ kW$

$N = 160\ rpm$

$\tau = 70\ MPa$

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the maximum torque transmitted in each revolution exceeds the torque by 20% :

So, $T = T + 20\%$ of T

$$\Rightarrow T = \left(1 + \frac{20}{100}\right) T$$

$$\Rightarrow T = (1 + 0.2) T$$

$$\Rightarrow T = 1.2\,T$$

$$P = \frac{2\pi N T}{60}$$

$$\Rightarrow 100 = \frac{2\pi \times 160 \times 1.2\,T}{60}$$

$$\Rightarrow T = \frac{100 \times 60}{2\pi \times 160 \times 1.2}$$

$$\Rightarrow \boxed{T = 4.97 \text{ kN-m}}$$

$$T = \frac{\pi}{16} \tau D^3$$

$$\Rightarrow 4.97 = \frac{\pi}{16} \times 70 \times 10^6 \times D^3$$

$$\Rightarrow D^3 = \frac{4.97 \times 16}{\pi \times 70 \times 10^6} \text{ mm}$$

$$\Rightarrow D^3 = 3.61 \times 10^{-7} \text{ mm}$$

$$\Rightarrow D = \sqrt[3]{3.61 \times 10^{-7}} \text{ mm}$$

$$\Rightarrow \boxed{D = 7.12 \times 10^{-3} \text{ mm}}$$

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* Torsional Equation:

$$\boxed{\frac{T}{J} = \frac{C\theta}{l}}$$

where, $T \rightarrow$ torque transmitted

$J \rightarrow$ polar moment of inertia

$C \rightarrow$ modulus of rigidity

$\theta \rightarrow$ angle of twist

$l \rightarrow$ length of the shaft

* Torsional Rigidity:

It is defined as "The torque required to produce a twist of one radian per unit length of the given shaft."

→ It is given by, $\boxed{T = CJ}$

where, $CJ \rightarrow$ torsional rigidity

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1) Find the maximum torque that can be applied safely to a shaft of 300 mm diameter. The angle of twist is 1.5° in length of 7.5m and the shear stress is not to exceed 42 N/mm^2. Take C = 84.4 kN/mm^2.

given:) $d = 300\ mm$

$C = 84.4\ kN/mm^2$

$= 84.4 \times 10^3\ N/mm^2$.

$\tau = 42\ N/mm^2$.

$l = 7.5\ m$

$= 7.5 \times 10^3\ mm$.

$\theta = 15°$

$= \frac{\pi}{180} \times 15°\ rad$.

$= 0.026\ rad$

$\left[\because \theta \rightarrow rad = \frac{\pi}{180} \right]$

$J = \frac{\pi}{32} d^4$

$\Rightarrow J = \frac{\pi}{32} \times (300)^4$

$\Rightarrow \boxed{J = 795.21 \times 10^6}$

$\therefore \frac{T}{J} = \frac{C\theta}{l}$

$\Rightarrow T = \frac{C\theta J}{l}$

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$$\Rightarrow T = \frac{84 \times 10^3 \times 0.026 \times 795.21 \times 10^7}{7.5 \times 10^3}$$

$$\Rightarrow \boxed{T = 231.56 \times 10^6 \text{ N-mm}}$$

② A bar of magnesium alloy 28 mm in diameter. was tested in a gauge length of 25 cm in tension and torsion. The tensile load of 5000 N produced an extension of 0.4 mm and a torque of 1250 Ncm produce a twist of 1.51°.

Determine the Young's Modulus (E), Bulk Modulus (K), Modulus of Rigidity (C) and the poisson's ratio (μ).

given:

$d = 28$ mm $= 2.8$ cm

$\Delta l = 0.4$ mm $= 0.04$ cm

$P = 5000$ N

$l = 25$ cm

$T = 1250$ N-cm

$\theta = 1.51°$

$= \frac{\pi}{180} \times 1.51°$ rad.

$= 0.026$ rad

(i) $\Delta l = \frac{Pl}{AE} \Rightarrow E = \frac{Pl}{A \Delta l}$

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$$\Rightarrow E = \frac{5000 \times 25}{\frac{\pi}{4}(2.8)^2 \times 0.04}$$

$$\Rightarrow \boxed{E = 507.50 \times 10^3 \ N/cm^2}$$

$$J = \frac{\pi}{32} d^4$$

$$\Rightarrow J = \frac{\pi}{32}(2.8)^4$$

$$\Rightarrow \boxed{J = 6.03 \ mm^4}$$

(ii) Modulus of rigidity (C):

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$\Rightarrow C = \frac{Tl}{J\theta}$$

$$\Rightarrow C = \frac{1250 \times 25}{6.03 \times 0.026}$$

$$\Rightarrow \boxed{C = 199.32 \times 10^3 \ N/cm^2}$$

(iii) Poisson's Ratio (μ or $\frac{1}{m}$):

$$E = 2C\left(1 + \frac{1}{m}\right)$$

$$\Rightarrow \frac{E}{2C} = 1 + \frac{1}{m}$$

$$\Rightarrow \frac{1}{m} \text{ or } \mu = \frac{E}{2C} - 1$$

$$\Rightarrow \mu = \frac{507.50 \times 10^3}{2(199.32 \times 10^3)} - 1$$

$$\Rightarrow \boxed{\mu = 0.27}$$

(iv) Bulk Modulus (K):

$$E = 3K\left(1 - \frac{2}{m}\right)$$

$$\Rightarrow E = 3K\left(1 - 2 \cdot \frac{1}{m}\right)$$

$$\Rightarrow E = 3K\left(1 - 2 \cdot \mu\right)$$

$$\Rightarrow E = 3K\left(1 - (2 \times 0.27)\right)$$

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$$\Rightarrow \quad E = 3K \times 0.46$$

$$\Rightarrow \quad K = \frac{E}{3 \times 0.46}$$

$$\Rightarrow \quad K = \frac{507.50 \times 10^3}{3 \times 0.46}$$

$$\Rightarrow \quad K = 367.75 \times 10^3 \ N/cm^2.$$

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Chapter 4

COLUMN AND STRUTS

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CHAPTER-7

COLUMNS AND STRUTS

* Columns:

→ It is a vertical member made up of brick RCC, steel, timber which is only subjected to compression.

→ It takes loads from either lintel or roof slab.

* Types of Columns:

Columns may be of 2 types:

① Short column
② Long column

① Short Column:

A column is said to be a short column if the ratio of effective length to its least lateral dimension is less than or equal to 12.

(OR)

A column is said to be a short column when the ratio of effective length to the least radius of gyration is less than 45. (i.e. slenderness ratio)

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(OR)

A column is said to be a short column when the length of the column is less as compared to its cross-sectional dimension.

② Long Column :

A column is said to be a long column if the ratio of effective length to its least lateral dimension is not less than 12.

(OR)

A column is said to be a long column if the ratio of effective length to the least radius of gyration is greater than 45 (i.e. slenderness ratio)

* Slenderness Ratio :

→ It is defined as "The ratio of effective length to least radius of gyration".

→ It is denoted by 'λ' (lambda) and mathematically,

$$\boxed{\lambda = \frac{L_{eff}}{K}}$$

where, K → least radius of gyration

L_{eff} → effective length.

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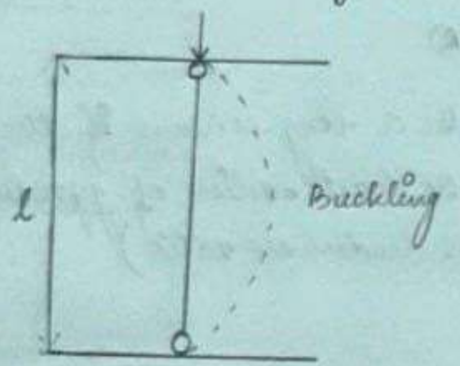
* **Effective length of a column :** (2 marks or 6 marks)

This is the distance between the successive points of zero moment.

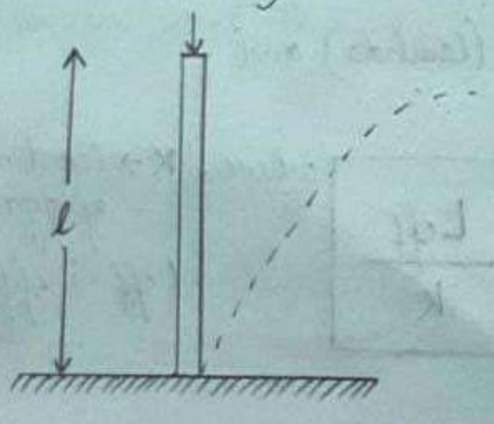
* **Effective length of column for various end condition :** $\boxed{\text{Actual length} = l}$

Case : 1 → Both ends hinged



$\boxed{L_{eff} = l}$

Case : 2 → One end fixed, one end free



$\boxed{L_{eff} = 2l}$

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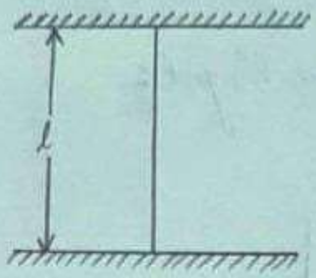
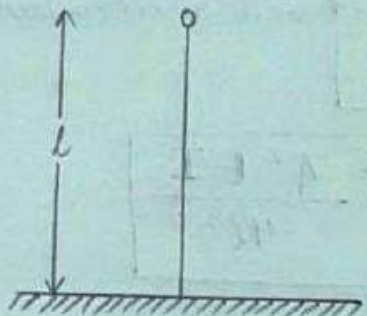
Case: 3 → Both ends fixed



$$\boxed{L_{eff} = \frac{l}{2}}$$

Case: 4 → One end fixed, one end hinged



$$\boxed{L_{eff} = \frac{l}{\sqrt{2}}}$$

* Euler's Theory of Long Column: 29.12.21

Euler's Theory of long column is given by:

$$\boxed{P = \frac{\pi^2 EI}{(l_{eff})^2}}$$

where, P → crippling load
E → Young's Modulus
I → Moment of Inertia
l_{eff} → effective length

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→ Euler's theory of long column is based on different end conditions of column are given below:

Case 1: When both ends are hinged;

$$\boxed{l_{eff} = l}$$

$$\boxed{P = \frac{\pi^2 EI}{l^2}}$$

Case 2: When one end fixed and another end is free; $\boxed{l_{eff} = 2l}$

$$P = \frac{\pi^2 EI}{(2l)^2} \Rightarrow \boxed{P = \frac{\pi^2 EI}{4l^2}}$$

Case 3: When both ends are fixed;

$$\boxed{l_{eff} = \frac{l}{2}}$$

$$P = \frac{\pi^2 EI}{(l/2)^2} \Rightarrow P = \frac{\pi^2 EI}{\left(\frac{l^2}{4}\right)}$$

$$\Rightarrow \boxed{P = \frac{4\pi^2 EI}{l^2}}$$

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Case 4 : When one end is fixed and another is hinged ; $\boxed{l_{eff} = \frac{l}{\sqrt{2}}}$

$$P = \frac{\pi^2 EI}{\left(\frac{l}{\sqrt{2}}\right)^2} \Rightarrow P = \frac{\pi^2 EI}{\left(\frac{l^2}{2}\right)}$$

$$\Rightarrow \boxed{P = \frac{2\pi^2 EI}{l^2}}$$

Questions :

① A mild steel tube 4m long, 30mm internal diameter and 4mm thick is used as a column with both ends hinged. Find the collapsing load.

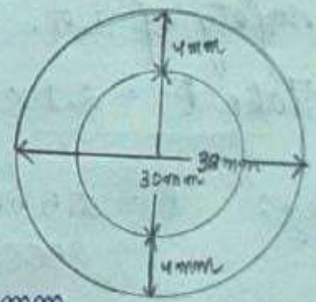
Take $E = 2.1 \times 10^5\ N/mm^2$.

given :) $l = 4m$
$= 4000mm = 4 \times 10^3\ mm$

(d) internal diameter $= 30mm$

$E = 2.1 \times 10^5\ N/mm$

thickness $= 4mm$



(D) external diameter $= 30mm + 4mm + 4mm$
$= 38mm$

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}\left((38)^4 - (30)^4\right) \Rightarrow I = 62.59 \times 10^3\ mm^4$$

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$$\therefore P = \frac{\pi^2 EI}{l^2}$$

$$\Rightarrow P = \frac{\pi^2 \times 2.1 \times 10^5 \times 62.59 \times 10^3}{(4 \times 10^3)^2}$$

$$\Rightarrow \boxed{P = 8.1 N}$$

$$\Rightarrow \boxed{P = 8.1 \times 10^{-3} kN}$$

② A strut 2.5m long is 60mm in diameter, One end of strut is fixed while its other end is hinged.
Find the safe compressive load for the member using Euler's formula, allowing a factor of safety of 3.5.
Take $E = 2.1 \times 10^5 \, N/mm^2$

given: $l = 2.5 \, m$
$= 2500 mm$
$d = 60 mm$

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64} (60)^4$$

$$\Rightarrow I = 636.17 \times 10^3 \, mm^4$$

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$$P = \frac{2\pi^2 EI}{l^2}$$

$$\Rightarrow P = \frac{2\times\pi^2 \times 2.1\times 10^5 \times 636.17 \times 10^3}{(2500)^2}$$

$$\Rightarrow \boxed{P = 421.93 \times 10^3 \, N}$$

$$\Rightarrow \boxed{P = 421.93 \, kN}$$

$\therefore$ Safe Permissible load

$$= \frac{P}{\text{factor of safety}} = \frac{421.93}{3.5}$$

$$\Rightarrow \boxed{SPL = 120.55 kN}$$

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③ A solid circular compressive member 50mm in diameter is to be replaced by a hollow circular section of the same material.
Find the size of the hollow section if the internal diameter is 0.6 times the external diameter.

given:) A solid circular compression member of diameter = 50mm

Let the external diameter of the hollow circular section = 'x' mm

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then we have the internal diameter of the hollow circular section = $0.6 \times x$ mm

Therefore due to same moment of inertia:

$$I_{solid} = I_{hollow}$$

$$\Rightarrow \cancel{\frac{\pi}{64}}(d)^4 = \cancel{\frac{\pi}{64}}(D^4 + d^4)$$

$$\Rightarrow d^4 = D^4 - d^4$$

$$\Rightarrow (50)^4 = x^4 - (0.6x)^4$$

$$\Rightarrow (50)^4 = x^4 - 0.1296\, x^4$$

$$\Rightarrow (50)^4 = x^4(1 - 0.1296)$$

$$\Rightarrow \frac{(50)^4}{(1-0.1296)} = x^4$$

$$\Rightarrow x^4 = \frac{(50)^4}{1-0.1296}$$