R. K. Institute of Engineering and Technology

Approved by AICTE, New Delhi and Affiliated SCTE&VT/BPUT, Odisha

LECTURE NOTE

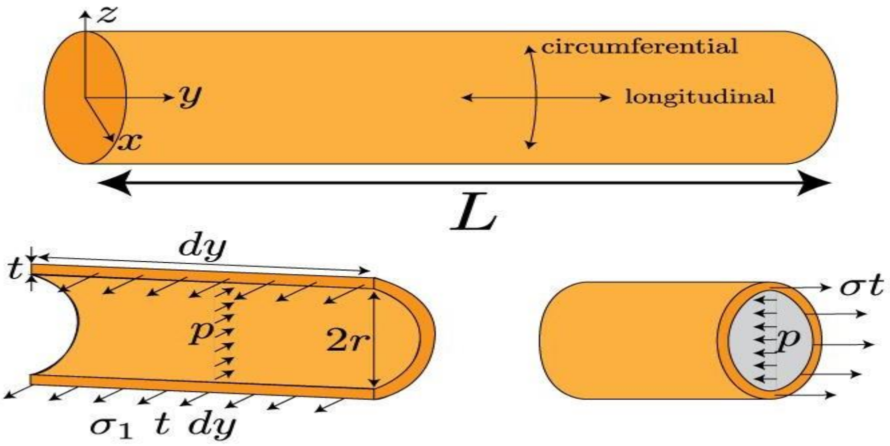
ON

STRENGTH OF MATERIAL

(DIPLOMA 3rd SEM)

DEPARTMENT OF MECHANICAL ENGINEERING

PREPARED BY: Er. SATYABRATA NAYAK
Lecturer in Department of Mechanical Engineering
RKIET,Niali,Cuttack
Mail ID-: satyaautomech.pupun@gmail.com



R.K. INSTITUTE OF ENGINEERING AND TECHNOLOGY

NIALI, CUTTACK

(Approved by AICTE)

Syllabus

1.1 Simple stress & strain

- **1.2** Types of load, stresses & strains,(Axial and tangential) Hooke's law, Young's modulus, bulk modulus, modulus of rigidity,
  Poisson's ratio, derive the relation between three elastic constants,
- **1.3** Principle of super position, stresses in composite section
- **1.4** Temperature stress, determine the temperature stress in composite bar (single core)
- **1.5** Strain energy and resilience, Stress due to gradually applied, suddenly applied and impact load
- **1.6** Simple problems on above.

2.1 Thin cylinder and spherical shell under internal pressure

- **2.2** Definition of hoop and longitudinal stress, strain
- **2.3** Derivation of hoop stress, longitudinal stress, hoop strain, longitudinal strain and volumetric strain
- **2.4** Computation of the change in length, diameter and volume
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- **3.4** Location of principal plane and computation of principal stressand Maximum shear stress using Mohr's circle

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- **4.2** Types of beam and load
- **4.3** Concepts of Shear force and bending moment
- **4.4** Shear Force and Bending moment diagram and its salient features illustration in cantilever beam, simply supported beam and over hanging beam under point load and uniformly distributed load

5.1 Theory of simple bending

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- **5.4** Solve simple problems.

6.1 Combined direct & bending stresses

- **6.2** Define column
- **6.3** Axial load, Eccentric load on column,
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- **6.5** Buckling load computation using Euler's formula (no derivation) in Columns with various end conditions
- **6.6** Buckling load computation using Euler's formula (no derivation) in Columns with various end conditions

7.1 Torsion

- 7.1 Assumption of pure torsion
- 7.2 The torsion equation for solid and hollow circular shaft
- 7.3 Comparison between solid and hollow shaft subjected to pure torsion

Chapter- 01
Simple Stress & Strain

➢ Strength of Material:

It is defined as the subject in mechanical engineering which deals with the study of behavior of structural and Machine members under action of external force. Due to application of external forces on the members, they deforms and leads failure.

It calculates the internal forces, stresses induced in body and their resistance against its deformation.

➢ Strength:

It is defined as the ability of to resist its failure and behavior under action of external forces. The external forces are called Load.

In our practical life we have come across Materials classified basically as Rigid, Plastic & Elastic material

i. Rigid Material: It does not undergoes any deformation under the application of external forces
ii. Plastic Material: It undergoes a continuous deformation during period of loading and deformation can't be recovered during removal of load on the material.
iii. Elastic Material: It under goes deformation under action of loading and deformation disappears on the removal of load

In practical cases no material is absolutely elastic nor plastic nor rigid. But in this case we will study about elastic materials under action external loads.

➢ Load:

It is defined as the external force applied on the body. It is characterized by its magnitude and Direction

Unit: it is expressed in N, KN, MN, and GN

Basically there are two types of load

i. Direct Load /Axial load
   a) Tensile Load, b) Compressive Load
ii. Tangential Load (Shear force)

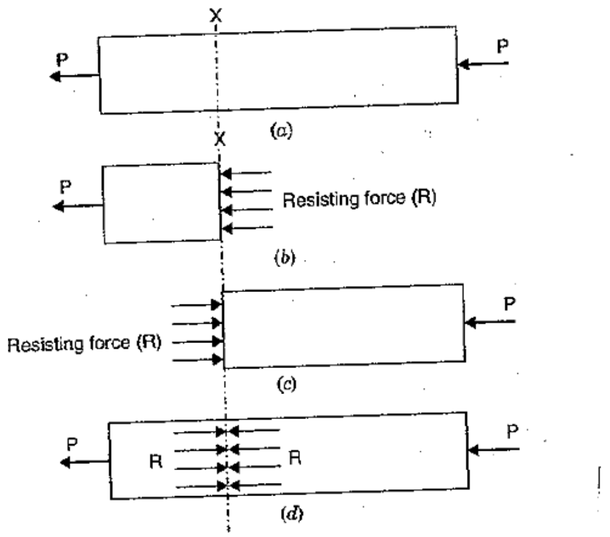
➢ Stress:

It is defined as the force of resistance offered by body against the deformation under action of external load

It may be defined as the intensity of resistance per unit area of cross-section of body.

It is denoted by symbol $'\sigma'$

It is used to mean intensity of stress.



$$Intensity\ of\ Stress\ ,\sigma = \frac{internal\ resistance\ force}{Area\ of\ Cross-section} = \frac{R}{A}$$

$$= \frac{P}{A}$$

$$\boldsymbol{\sigma = \frac{P}{A}}$$

where

$P - Load\ applied\ on\ body\ ,A - Area\ of\ Cross-section$

Unit: it is expressed in $N/_{mm^2}$, $Pascal(Pa)$

$$1\ Pascal(Pa) = \frac{N}{m^2}$$

$$1\ MPa = 10^6\ \frac{N}{m^2}$$

$$1GPa = 10^9\ \frac{N}{m^2}$$

➢ Types of Stress:

It is classified as following (on the basis of direction of load acting on body)

i. Direct Stress/Normal Stress/Axial Stress
   a) Tensile Stress , b) Compressive Stress
ii. Shear Stress/Tangential Stress

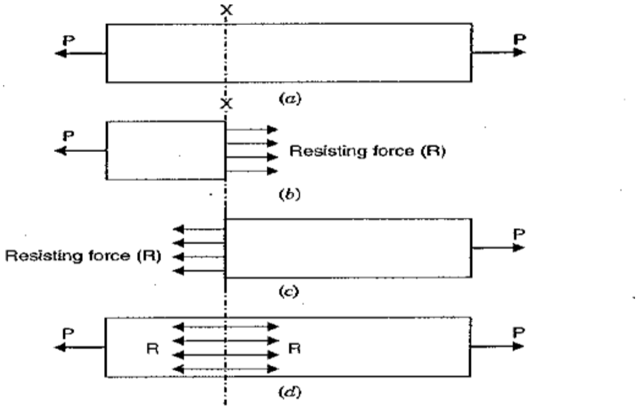
➢ Tensile Stress:

Whenever a body is subjected to two equal pulls acting at its two ends of body then tensile stress is induced.
It may be also defined as the ratio of tensile load to the area of cross-section of body.
It is denoted by Symbol $'\sigma_t'$
Mathematically

$$\text{Tensile stress} = \frac{\text{tensile load}}{\text{area of Cross} - \text{section}} = \frac{P}{A}$$

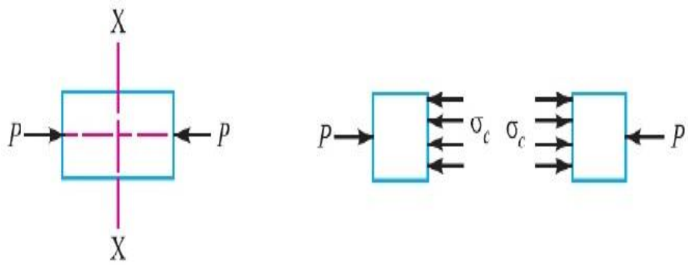
$$\sigma_t = \frac{P}{A}$$

Unit: it is expressed in $N/_{mm^2}$, $Pascal(Pa)$



➢ Compressive Stress:

Whenever a body is subjected to two equal pushes acting at its two ends of body then tensile stress is induced.
It may be also defined as the ratio of Compressive load to the area of cross-section of body.
It is denoted by Symbol $'\sigma_c'$
Mathematically



$$\text{Compressive stress} = \frac{\text{Compressive load}}{\text{area of Cross} - \text{section}} = \frac{P}{A}$$

$$\sigma_c = \frac{P}{A}$$

Unit: it is expressed in $N/_{mm^2}$, $Pascal(Pa)$

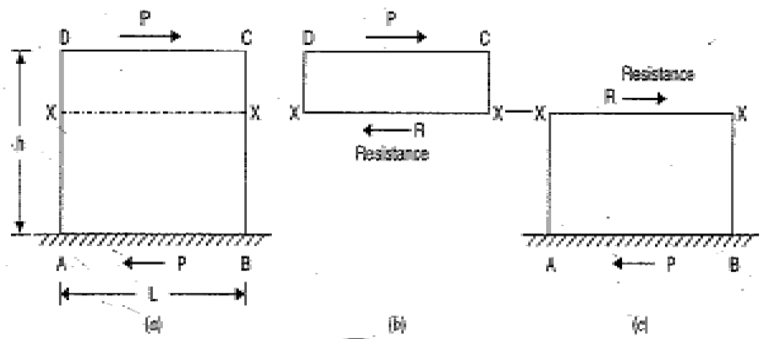
➢ Shear Stress:

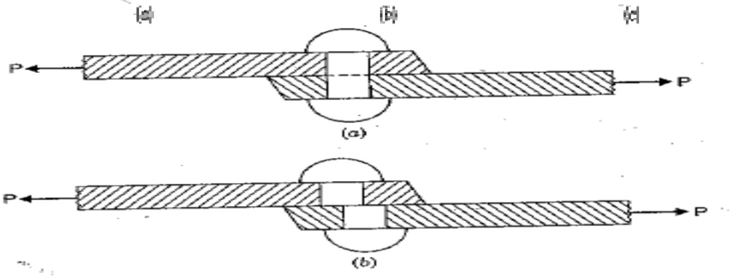
Whenever a body is subjected to tangential force on its surface, then shear stress is produced in it.
It is also defined as the ratio of tangential force/Shear force to shear area of body.
It is denoted by $'\tau'$
Mathematically

$$Shear\ stress = \frac{Shear\ force}{Shear\ area} = \frac{P}{A}$$

$$\tau = \frac{P}{A}$$

Unit: it is expressed in $N/_{mm^2}$, $Pascal(Pa)$





➢ Shear Force:

It may be defined as the force which is acting parallel to the surface of body. (Line of action of force is parallel with the surface of body)

Or it is the force which is acting tangentially along a surface is called shear force.

➢ Strain:

It is defined as the deformation per unit length.

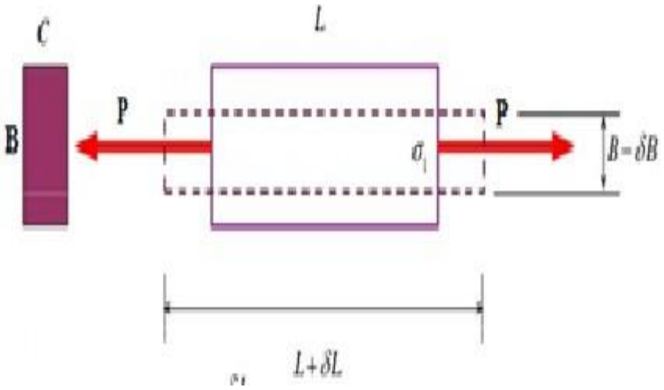
Whenever a body is subjected to external force then there is a deformation takes place in body i.e. length of body changes from its original length.

It is denoted by symbol 'e'

Mathematically

$$Strain \quad e = \frac{change\ in\ length(\delta l)}{original\ length(l)} = \frac{\delta l}{l}$$

Unit: it is unit less.



Q. What do you mean by stress & Strain? **[2018 S (N)]**

Ans.:

Stress: It may be defined as the intensity of resistance per unit area of cross-section of body.

$$Intensity\ of\ Stress, \sigma = \frac{internal\ resistance\ force}{Area\ of\ Cross - section} = \frac{R}{A} = \frac{P}{A}$$

$$\boldsymbol{\sigma = \frac{P}{A}}$$

$where\ \ P - Load\ applied\ on\ body\ ,\ A - Area\ of\ Cross - section$

Unit: it is expressed in $^{N}/_{mm^2}$, $Pascal(Pa)$

Strain: It is defined as the deformation per unit length. It is denoted by symbol 'e'

Mathematically $\quad Strain \quad e = \dfrac{change\ in\ length(\delta l)}{original\ length(l)} = \dfrac{\delta l}{l}$

Unit: it is unit less.

➢ Types of Strain:

i. Direct Strain /Normal Strain
   a) Tensile Strain b) Compressive Strain

ii. Shear Strain

➢ Tensile Strain:

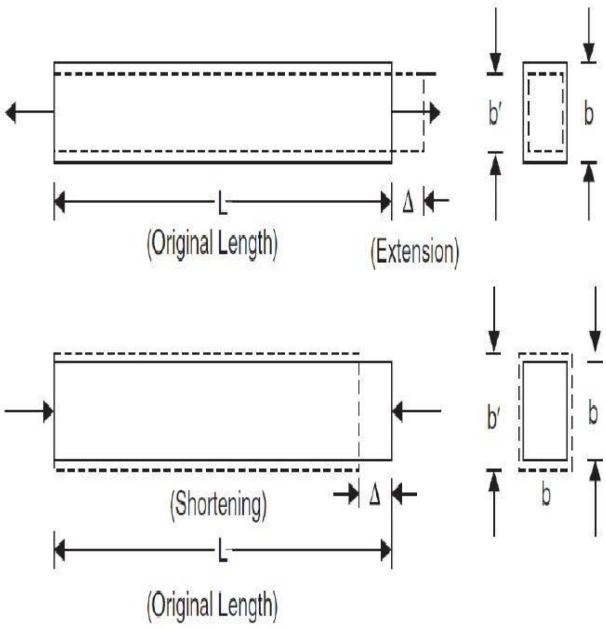
It is defined as the ratio of increase in length to original length of body, under action of tensile load. It is denoted by symbol $'e_t'$

Mathematically,

$$\text{Tensile strain} = \frac{\text{increase in length}}{\text{origial length}} = \frac{\delta l}{l},$$

$$e_t = \frac{\delta l}{l}$$

Unit: it is unit less



➢ Compressive Strain:

it is defined as the ratio of decrease in length to original length of body, under action of Compressive load. It is denoted by symbol $'e_c'$

Mathematically,

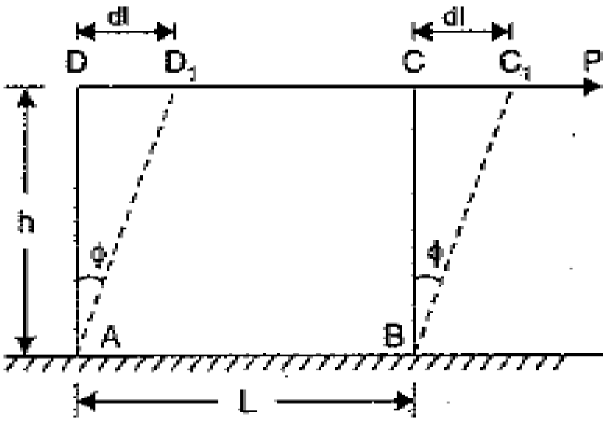
Tensile strain $= \dfrac{\text{decrease in length}}{\text{origial length}} = \dfrac{\delta l}{l}$,

$$e_c = \frac{\delta l}{l}$$

Unit: it is unit less

- ## Shear Strain :

It is the ratio of transverse displacement to distance from lower face as shown in fig.



Let's consider a body ABCD whose Face AB is fixed and face CD is free. Now the body is subjected to shear force at its face CD. Now the body will be distorted to ABC_1D_1 from ABCD with an angular deformation $'\varphi'$

As shown in fig. let's consider

$l -$ *original length of body*

$dl -$ *change in length of body*

$h -$ *height of body*

$P -$ *Tangential load acting on body*

We have horizontal displacement of upper face of body i.e. $CD - C_1D_1$ having deformation 'dl'. The side AD transverse along angular displacement as AD'

Now $shear\ strain = \dfrac{transverse\ \ Displacement}{distance\ of\ layer\ from\ lowr\ face} = \dfrac{dl}{h}$

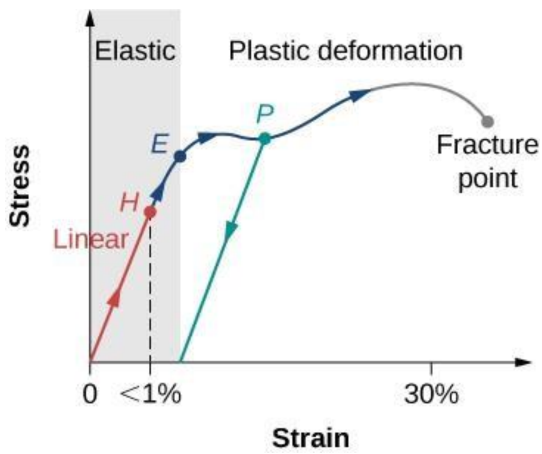
From triangle Δ ADD' shear strain

$$\tan\varphi = \frac{DD_1}{AD} = \frac{dl}{h}, \text{ Very small angle of } \tan\varphi = \varphi,$$

Shear strain $\varphi = \dfrac{dl}{h}$

- ## Elastic Limit:

Definition: it is defined as the maximum value of load or stress applied or stress generated up to which the material shows property of Elasticity.



i. Every material exhibits the property of elasticity for which it regains its original shape and size when intensity of stress induced is within elastic limit, called elastic limit.
ii. Beyond elastic limit material deforms continuous under action of loading , the deformation is permanent in nature and at this stage is called plastic and below the elastic limit point the material exhibits elastic property which enables the material to regain its original shape and size after removal of load.

- ## Hooke's law:

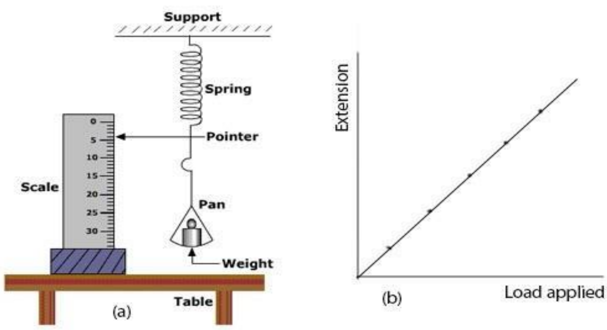
Statement: It states that whenever a material is loaded within elastic limit, stress is directly proportional to strain

Mathematically $\quad Stress \propto Strain$

$\sigma \propto e \Rightarrow \sigma = E.e$

$\Rightarrow \boldsymbol{E} = \dfrac{\boldsymbol{\sigma}}{\boldsymbol{e}}$

where $E -$ *Young's modulus of Elasticity*



Q. State Hooke's law (How stress & strain are related within elastic limit ?

Ans. **[2011(w), 2014w, 2016(O&N), 2009, 2017 W (N)]**

Statement: It states that whenever a material is loaded within elastic limit, stress is directly proportional to strain

Mathematically $Stress \propto Strain$ $\sigma \propto e \Rightarrow \sigma = E.e$

$\Rightarrow \boldsymbol{E} = \frac{\boldsymbol{\sigma}}{\boldsymbol{e}}$ $where\ E - Young's\ modulus\ of\ Elasticity$

Q. **What is Young's modulus of Elasticity?** [2017 W (N)]

- Young's Modulus of elasticity:

Whenever a material is axially loaded within elastic limit, then the ratio of intensity of direct stress to corresponding Strain is known as Modulus of Elasticity of material.

It is denoted by symbol 'E'

Mathematically

$Young's\ Modulus\ of\ Elasticity$

$$E = \frac{direct\ Stress(tensile\ or\ Compressive)}{direct\ strain(Tensile\ or\ Compressive\ )} = \frac{\sigma}{e}$$

$$\boldsymbol{E} = \frac{\boldsymbol{\sigma}}{\boldsymbol{e}}$$

Unit: it is expressed in $N/_{mm^2}$, $Pascal(Pa)$

Q. 1

An elastic rod 25 mm in diameter, 200 mm long extends by 0.25 mm under a tensile load at 40 KN. Find the intensity of stress, the strain and the elastic modulus for the material of the rod.

Ans.:

Given Data:

$d = 25\ mm$

$l = 200\ mm$

$\delta l = 0.25$

$P = 40\ KN$

$Area\ of\ crosssection\ of\ Rod,$ $A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 25^2 = 490.87\ mm^2$

$Stress\ induced\ in\ rod,$ $\sigma = \frac{P}{A} = \frac{40 \times 10^3}{490.87} = 81.48\ N/_{mm^2}$

$Strain\ induced\ in\ Rod,$ $e = \frac{\delta l}{l} = \frac{0.25}{200} = 0.0013$

$$Young's\ Modulus\ of\ Elasticity\ \ E = \frac{\sigma}{e} = \frac{81.48}{0.0013} = 65.16\ N/_{mm^2}$$

Q.2

A tensile load of 60 KN applied axial on a cylinder bar of 10 cm. What is the tensile stress on a section perpendicular to the axis of bar

Ans.:

Given data:

$P = 60\ KN$

$d = 10\ cm = 100mm$

$\sigma = ?$

$Area\ of\ Crossection\ of\ Bar\ , A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 100^2 = 7853.98\ mm^2$

$Stress\ induced\ in\ rod,$ $\sigma = \frac{P}{A} = \frac{60 \times 10^3}{7853.98} = 7.63\ N/_{mm^2}$

➢ Modulus of Rigidity:

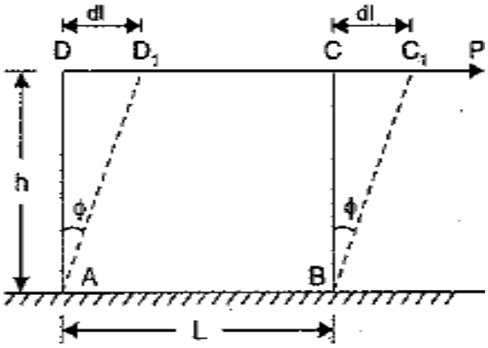
In case of shear loading on material with in elastic limit, the ratio of shear stress to corresponding Shear strain is called Modulus of Rigidity. It is denoted by 'C', 'G', 'n'

Mathematically:

$$Modulus\ of\ Rigidity, G = \frac{shear\ stress}{Shear\ strain} = \frac{\tau}{\varphi}$$

$$G = \frac{\tau}{\varphi}$$

Unit: it is expressed in $N/_{mm^2}$, $Pascal(Pa)$



➢ Bulk Modulus of Elasticity:

When a body is subjected to three like and equal direct stresses along three mutually perpendicular directions .then the ratio of direct stress to volumetric strain gives a constant for a given material, when the deformation is within Elastic limit the ratio is called Bulk Modulus of Elasticity.

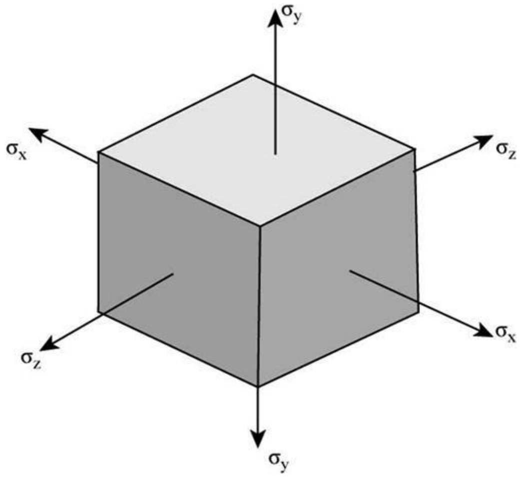
It is denoted by Symbol 'K'

Let's consider a cube is subjected to direct stresses along three mutually perpendicular direction.

Mathematically

$$Bulk\ Modulus, K = \frac{normal\ stress}{volumetric\ strain} = \frac{\sigma_n}{e_v}$$

$$\boldsymbol{K = \frac{\sigma_n}{e_v}}$$



Unit: it is expressed in $N/_{mm^2}$, $Pascal(Pa)$

➢ Volumetric Strain:

It is defined as the ratio of change in volume to original volume of body

It is denoted by $'e_v'$

Mathematically

$Volumetric\ strain$

$$e_V = \frac{Change\ in\ Volume}{Original\ Volume} = \frac{\delta V}{V}$$

$$\boldsymbol{e_v = \frac{\delta V}{V}}$$

Unit: it is unit less.

➢ Longitudinal Strain:

It is defined as the ratio of change in length to original length of body when it is subjected to axial stresses (tensile or Compressive Stress).

It is denoted by 'e_l'

Mathematically

$longitudinal\ strain$

$$e_l = \frac{Change\ in\ length}{Original\ length} = \frac{\delta l}{l}$$

$$\boldsymbol{e_l = \frac{\delta l}{l}}$$

Unit: it is unit less

Lateral Strain:

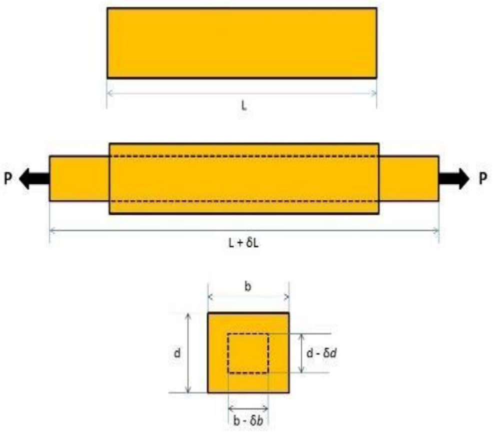
It is defined as the ratio of change in lateral dimension to original lateral dimension.it is denoted by symbol $'e_{la}'$

In case of rectangular bar during axial loading with change in length, their width and depth also undergoes change.

Now lateral strain for Rectangular bar

$$e_{la} = \frac{change\ in\ width\ or\ depth}{original\ width\ or\ depth} = \frac{\delta b}{b} or \frac{\delta d}{d}$$

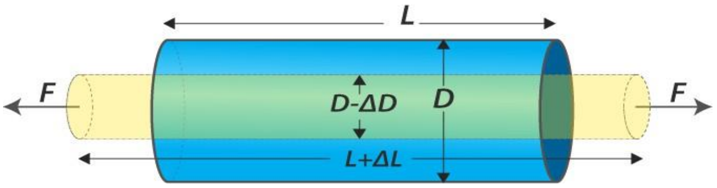
$$e_{la} = \frac{\delta b}{b} or \frac{\delta d}{d}$$



In case of cylindrical body, when it is subjected to axial load along its length, its diameter undergoes change.

Now lateral strain for cylindrical bar

$$lateral\ Strain\ e_{la} = \frac{Change\ in\ Diameter}{Original\ Diameter} = \frac{\delta d}{d}$$



Unit: it is unit less

Q. Define poison's ratio. **[2017W, 2014, 2012, 2011, 2010, 2009]**

➢ Poisson's Ratio:

When the deformation of the member is within elastic limit, it is found that the ratio of lateral strain to the longitudinal strain is a constant for a given material. The ratio is known as Poisson's ratio.

It is denoted by symbol 'μ'

Now Poisson's ratio

$$\mu = \frac{Lateral\ strain}{Longitudinal\ strain} = \frac{e_{la}}{e_l}$$

$$\mu = \frac{\delta d/_d}{\delta l/_l} \ldots\ldots\ldots\ldots..For\ cylindrical\ body$$

$$\mu = \frac{\delta b/_b}{\delta l/_l} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.for\ rectangular\ body$$

It is unit less.

Q. A concrete cylinder of dia. 150mm and length 300mm when subjected to an axial compressive load of 240KN resulted in an increase of dia. by 0.127mm and a decrease in length of 0.28mm. compute the value of Poisson's ratio and modulus of Elasticity.

Ans.:

Given Data:

$d = 150\ mm$

$l = 300\ mm$

$P = 240\ KN = 240 \times 10^3\ N$

$\delta l = 0.28\ mm$

$\delta d = 0.127$

$\mu, E = ?$

$$Area\ of\ concrete\ cylinder, \qquad A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 150^2 = 17671.45\ mm^2$$

$$Compressive\ stress\ induced\ in\ Concrete\ cylinder\ \sigma = \frac{P}{A} = \frac{240 \times 10^3}{17671.45} = 13.58\ ^N/_{mm^2}$$

$$lateral\ strain, \qquad e_{la} = \frac{\delta d}{d} = \frac{o.127}{150} = 0.0008$$

$$longtudinal\ Strain,\quad e_l = \frac{\delta l}{l} = \frac{0.28}{300} = 0.0009$$

$$Poisson's\ Ratio\ \mu = \frac{e_{la}}{e_l} = \frac{0.0008}{0.0009} = \mathbf{0.88}$$

$$Modulus\ of\ elasticity\ E = \frac{\sigma}{e_l} = \frac{13.58}{0.0009} = \mathbf{15088.88}\ \mathbf{N/mm^2}$$

Q. Derive the relation between three elastic constants. **[2009, 2010, 2013 & 2018]**

Ans.:

- Relationship between Modulus of Elasticity and Modulus of Rigidity:

Let's consider a cube ABCD of unit thickness as shown in figure. Let 'l' be the each side of cube.

Shear stress is applied to all sides of cube, considering shear stress acting along the top surface of cube which causes the deformation of diagonal and forming the new cube as ABC'D'.

Let's consider the deformation takes place along the diagonal 'BD' and AC which are changed to BD' and AC' respectively.

The deformation takes place through angular deformation $'\emptyset'$

Considering deformation of diagonal BD-BD'

Now shear strain along diagonal BD

$$\epsilon_{BD} = \frac{BD' - BD}{BD} = \frac{D'D''}{BD} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (i)$$

Now drop a perpendicular form point D as DD'' $\perp^{ar}$ BD'

From $\Delta\ DD'D''$ $\cos\emptyset = \frac{D'D''}{DD'} \cdots\cdots\cdots (ii)$

Similarly from Δ ADB $BD = \sqrt{2}\ AD$

$\Delta\ DD'D'' \sim \Delta$ ADB for very small angular deformation $\cos\emptyset = \cos 45°$

Now from equation-ii

$$\cos\emptyset = \frac{D'D''}{DD'} \cdots\cdots\cdots (ii)$$

$$\cos 45° = \frac{D'D''}{DD'} \Rightarrow D'D'' = D'D\cos 45° \cdots\cdots\cdots \text{(iii)}$$

Now putting value of BD & D'D form equation (ii) and (iii) on equation- (i)

$$\epsilon_{BD} = \frac{BD' - BD}{BD} = \frac{D'D''}{BD} = \frac{D'D\cos 45°}{\sqrt{2}\ AD} = \frac{\emptyset}{2}$$

$$\epsilon_{BD} = \frac{\emptyset}{2} \cdots\cdots\cdots (iv)$$

Basically whenever strain in BD takes place there will be another lateral strain along the AC take place.

Total strain in BD = $\frac{\tau}{E} + \mu\ \frac{\tau}{E} = \frac{\tau}{E}(1+\mu) \quad \cdots\cdots\cdots (v)$

We have Modulus of rigidity $G = \frac{\tau}{\emptyset} = \frac{\tau}{2\emptyset} \Rightarrow \emptyset = \frac{\tau}{G} \cdots\cdots (vi)$

Putting value of $'\emptyset'$ from eqn. (vi) on equation (iv)

Equating both equations (v) &(iv)

$$\epsilon_{BD} = \frac{\emptyset}{2} = \frac{\tau}{E}(1+\mu)$$

$$\Rightarrow \frac{\tau}{2G} = \frac{\tau}{E}(1+\mu)$$

$$\Rightarrow \boldsymbol{E = 2G(1+\mu)}$$

Q. Derive the relation between modulus of elasticity and Bulk modulus. [2017s]

➢ Relationship between Young's Modulus Elasticity & Bulk Modulus (E & K)

Let's consider a cube of ABCDEFGH of unit thickness and side 'a',is subjected to equal and like direct tensile stresses along the three perpendicular direction as shown in diagram.

Let

$\sigma_n - Normal\ stress\ acting\ on\ cube$

$a\ - length\ of\ each\ side\ of\ cube$

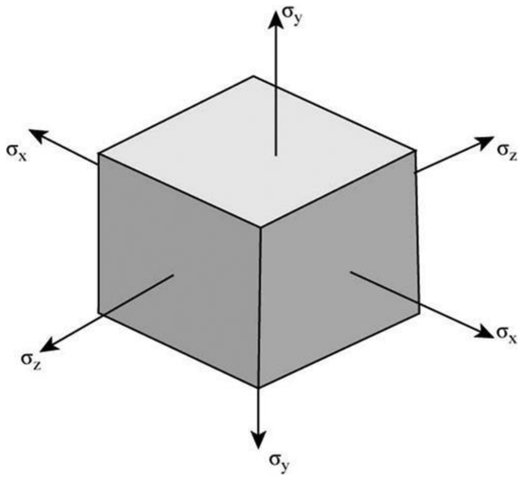
$V -\ Original\ volume\ of\ \ cube$

$\delta V - chnage\ in\ volume\ of\ body$

$\delta a - change\ in\ length\ of\ body$

$E - young's\ Modulus\ of\ elasticity\ for\ material\ of\ cube$

$\mu - poisson's\ ratio$



Now strain in' HG' due to normal stress on face ADEH and BC FG

$$e_{HG} = \frac{\sigma_n}{E}$$

Strain in 'HG 'due to normal stresses on face EFGH and ABCD

$$e_{HG} = -\mu\frac{\sigma_n}{E}$$

Strain in 'HG' due to normal stress face CDEF & ABGH

$$e_{HG} = -\mu\frac{\sigma_n}{E}$$

Total strain in 'HG'

$$e_{HG} = \frac{\sigma_n}{E} - \mu\frac{\sigma_n}{E} - \mu\frac{\sigma_n}{E} = \frac{\sigma_n}{E}(1-2\mu) \ldots\ldots\ldots\ldots\ldots\ldots.(i)$$

We know that for a cube side 'a'

Volume of Cube $V = a^3$

$change\ in\ volume\ of\ cube\ \delta V = 3a^2\delta a$

$$now\ volumatric\ Strain\ \ e_v = \frac{\delta V}{V} = \frac{3a^2\delta a}{a^3} = 3\frac{\delta a}{a} = 3e_{HG}$$

$$e_v = 3e_{HG} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

Now equating both equation (i) and (ii)

$$e_{HG} = \frac{\sigma_n}{E}(1-2\mu) \ldots\ldots\ldots\ldots\ldots.(i)$$

$$e_v = 3e_{HG} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

$$\frac{\sigma_n}{E}(1-2\mu) = \frac{e_v}{3}$$

$$E = 3\frac{\sigma_n}{e_v}(1-2\mu) = \ 3K\ (1-2\mu)$$

$$\boldsymbol{E = 3K\ (1-2\mu)}$$

➢ Relationship between elastic constants E, G & K

We know that

$$E = 3K\ (1-2\mu)\ \ \Rightarrow\ \frac{E}{3K} = 1-2\mu$$

$E = 2G\,(1+\mu) \quad \Rightarrow \frac{E}{2G} = 1+\mu \;\Rightarrow \frac{E}{G} = 2+2\mu$

Solving equation (I) & (ii)

$$\Rightarrow \frac{E}{3K} + \frac{E}{G} = 1 - 2\mu + 2 + 2\mu$$

$$\Rightarrow \frac{G+3K}{3KG} = \frac{3}{E} \qquad \Rightarrow \boldsymbol{E = \frac{9KG}{3K+G}}$$

Q. State the relation between three elastic constant? [2009(w), 2016wn][2 marks]

Ans. The relation between three elastic E, G and K

$$\boldsymbol{E = \frac{9KG}{3K+G}}$$

Where E- Modulus of Elastic, K- Bulk Modulus of Elasticity, G- Modulus of Rigidity

Q.3

Find the Bulk Modulus of the material if the modulus of rigidity and modulus of elasticity are $\boldsymbol{0.5 \times 10^5 N/mm2}$ *and* $\boldsymbol{1.15 \times 10^5 N/mm2}$ respectively. Also find out the lateral Contraction of a round bar of dia. 50mm dia. and 3 m length when stretched by 2.5 mm.

Ans.:

Given data:

$G = 0.5 \times 10^5 \dfrac{N}{mm2}$

$E = 1.15 \times 10^5 \dfrac{N}{mm2}$

$d = 50\ mm$

$l = 3m = 3000mm$

$\delta l = 2.5\ mm$

$\delta d = ?$

$K = ?$

We have relation between E and G

$$E = 2G(1+\mu) \Rightarrow \frac{E}{2G} - 1 = \mu$$

$$\Rightarrow \frac{1.15 \times 10^5}{2 \times 0.5 \times 10^5} - 1 = \mu$$

$$\Rightarrow \boldsymbol{\mu = 0.15}$$

$$Linear\ strain\ induced\ in\ cylinder\ , e_l = \frac{\delta l}{l} = \frac{2.5}{3000} = 0.0008$$

$$\boldsymbol{e_l = 0.0008}$$

$$we\ have\ poisson's\ ratio\ \mu = \frac{e_{la}}{e_l} \Rightarrow e_{la} = \mu . e_l = 0.15 \times 0.0008 = 0.0001$$

$$\Rightarrow \boldsymbol{e_{la} = 0.0001}$$

$$Lateral\ Strain\ , e_{la} = \frac{\delta d}{d} = 0.0001 \Rightarrow \delta d = 0.0001 \times 50 = 0.0063$$

$$\Rightarrow \boldsymbol{\delta d = 0.0063\ mm}$$

$we\ have\ relation\ between\ E\ and\ K\ ,$

$$E = 3K\,(1-2\mu) \Rightarrow \boldsymbol{K} = \frac{\boldsymbol{E}}{\boldsymbol{3}(1-2\mu)} = \frac{1.15 \times 10^5}{\boldsymbol{3}(1 - 2 \times 0.15)} = 0.5476 \times 10^5\ {}^{N}/_{mm^2}$$

$$Bulk\ modulus\ of\ elasticity\ , \boldsymbol{K = 0.5476 \times 10^5\ {}^{N}/_{mm^2}}$$

Q. 4

A material has a young's modulus of 1.25X 10^5 N/MM^2 and poison's ratio of 0.25. Calculate the modulus at rigidity and the Bulk modulus.

Ans.:

Given data

$$E = 1.25 \times 10^5 \ {}^{N}/{}_{mm^2} \quad \mu = 0.25 \ \ G, K = ?$$

we have relation between E and G ,

$$E = 2G(1+\mu) \Rightarrow \frac{E}{2(1+\mu)} = G$$

$$\Rightarrow G = \frac{E}{2(1+\mu)} = \frac{1.25 \times 10^5}{2(1+0.25)} = 50000 \ {}^{N}/{}_{mm^2}$$

$$\therefore \ Modulus\ of\ Rigidity\ , \boldsymbol{G = 50000\ {}^{N}/{}_{mm^2}}$$

we have relation between E and K ,

$$E = 3K(1-2\mu) \Rightarrow \frac{E}{3(1-2\mu)} = K$$

$$\Rightarrow K = \frac{E}{3(1-2\mu)} = \frac{1.25 \times 10^5}{3(1-2\times 0.25)} = 83333.33 \ {}^{N}/{}_{mm^2}$$

$$\therefore \ Bulk\ Modulus\ , \boldsymbol{K = 83333.33\ {}^{N}/{}_{mm^2}}$$

Q.5

In an elastic body, Young's Modulus is 200 N/m^2, Lateral strain 0.2×10^{-4} **and longitudinal strain is** 0.2×10^{-4}**. Find Modulus of Rigidity and Bulk's Modulus?** **[2017 W (N)]**

Ans.

Given Data:

$E = 200 \ {}^{N}/{}_{m^2}$

$e_{la} = 0.2 \times 10^{-4}$

$e_l = 0.8 \times 10^{-4}$

We have

$$poission's\ ratio,\ \mu = \frac{e_{la}}{e_l} = \frac{0.2\times10^{-4}}{0.8\times10^{-4}} = 0.25 \qquad , \boldsymbol{\mu = 0.25}$$

we have relation between E and G ,

$$E = 2G(1+\mu) \Rightarrow \frac{E}{2(1+\mu)} = G$$

$$\Rightarrow G = \frac{E}{2(1+\mu)} = \frac{200}{2(1+0.25)} = 80.00 \ {}^{N}/{}_{m^2}$$

$$\therefore \ Modulus\ of\ Rigidity\ , \boldsymbol{G = 80.00\ {}^{N}/{}_{m^2}}$$

we have relation between E and K ,

$$E = 3K(1-2\mu) \Rightarrow \frac{E}{3(1-2\mu)} = K$$

$$\Rightarrow K = \frac{E}{3(1-2\mu)} = \frac{200}{3(1-2\times 0.25)} = 133.33 \ {}^{N}/{}_{m^2}$$

$$\therefore \ Bulk\ Modulus\ , \boldsymbol{K = 133.33\ {}^{N}/{}_{m^2}}$$

➢ Deformation of bar due to axial load.

$Let's consider\ a\ bar\ is\ subjected\ to$
$P - load\ applied\ on\ bar$
$l - length\ of\ bar$
$A - Area\ of\ crosssection\ of\ bar$
$E - Young's modulus\ of\ Elasticity\ for\ material\ of\ bar$
$\delta l - deformation\ of bar$
$e - Strain\ induced\ in\ bar$

$we\ know\ that\ yong's modulus\ E = \frac{\sigma}{e} = \frac{P/A}{\delta l/l}$

$\Rightarrow \delta l/l = \frac{P/A}{E}$ $\qquad \Rightarrow \boldsymbol{\delta l = \frac{Pl}{AE}}$ ………………………………….$Deformation\ of\ bar$

Q. **Two wires, one of steel and the other of copper, are of the same length and are subjected to the same tensile load. If the diameter of copper wire is 20 mm, find the diameter of the steel wire, if they are elongated by the same amount. Take E for steel as 200 G Pa and that for copper as 100G Pa.**

Ans.:

Given Data:

$E_S = 200\ GPa = 200 \times 10^3\ N/mm^2$

$E_{Cu} = 100\ GPa = 100 \times 10^3\ N/mm^2$

$d_{CU} = 20\ mm$

Now area of Copper wire $A_{Cu} = \frac{\pi}{4} \times d_{CU}{}^2 = \frac{\pi}{4} \times 20^2 = 314.15\ mm^2$

$Tensile\ load\ on\ steel\ and\ Cupper\ wire\ are\ equal \quad P_S = P_{Cu}$

$length\ of\ both\ wires\ are\ equal\ \ l_S = l_{Cu}$

Since the two wire are subjected to same tensile load the deformation of both will be equal

$$\delta l_S = \delta l_{Cu}$$

$\Rightarrow \frac{P_S \times l_S}{A_S \times E_S} = \frac{P_{Cu} \times l_{Cu}}{A_{Cu} \times E_{Cu}} \quad \Rightarrow A_{Cu} \times E_{Cu} = A_S \times E_S$

$\Rightarrow 314.15 \times 100 \times 10^3 = A_S \times 200 \times 10^3$

$\Rightarrow \frac{314.15 \times 100 \times 10^3}{200 \times 10^3} = A_S$

$\Rightarrow \boldsymbol{A_S = 157.07\ mm^2}$

$\boldsymbol{A_S} = \frac{\pi}{4} \times d_s{}^2 = \boldsymbol{157.07}$

$\Rightarrow d_s{}^2 = \frac{4 \times \boldsymbol{157.07}}{\pi}$

$\Rightarrow d_s = \sqrt{\frac{4 \times \boldsymbol{157.07}}{\pi}} = 14.14\ mm$

$\therefore Diameter\ of\ steel\ wire, \qquad \boldsymbol{d_s = 14.14\ mm}$

➢ Principle of Super position:

Sometimes a body is subjected to a number of forces acting on its outer edges as well as at some other sections along the length of body ,in such case the forces are split up and their effect are considered on individual sections. The resulting deformation of body is equal to the algebraic sum of deformation of individual sections. Such a principle of finding out the resultant deformation is called principle of super position.

The relation for resulting deformation of uniform Cross section Bar may be modified as

$$\delta l = \frac{1}{AE}(P_1 l_1 + P_2 l_2 + P_3 l_3)$$

where

$P_1, P_2, P_3 -$ *Loads acting on induvidual sections of bar as* 1, 2, 3 *Respectively*

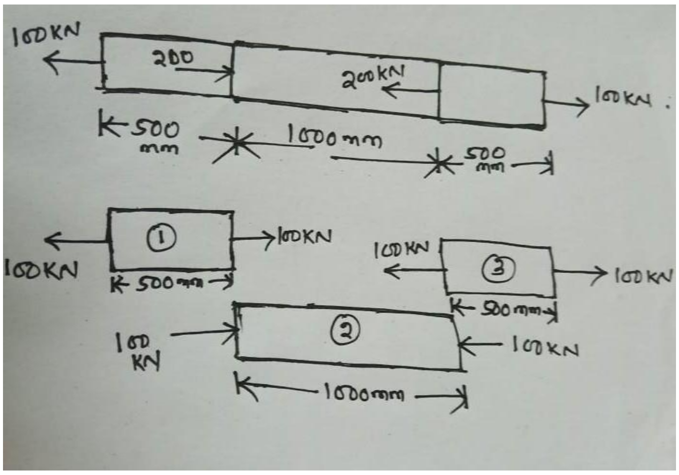
$l_1, l_2, l_3 -$ *length of induvidual sections of bar as* 1, 2, 3 *respectively*

Q.4

Find the Total Deformation in the Bar having diameter 25 mm and Modulus of Elastic for material of bar as $\boldsymbol{E = 2 \times 10^5 MPa}$ from following figure. [2017 W (N)]

Ans.

Given Data:



Diameter of bar d = 25 mm

Young's Modulus $E = 2 \times 10^5 MPa = 2 \times 10^5 \ {}^{N}/_{mm^2}$

$$Cross - Sectional\ Area\ of\ Bar\ A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 25^2$$

$$= 490.87\ mm^2$$

Splitting of forces and body into three Parts

Now we have

$P_1 = 100 \times 10^3\ N\ (Pull),\ \ l_1 = 500\ mm,$

$P_2 = -\ 100 \times 10^3\ N (push), l_2 = 1000\ mm$

$P_3 = 100 \times 10^3\ N (pull), l_3 = 500\ mm$

Applying principle of Super Position theorem

$$Total\ deformation\ of\ Bar\ \delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \frac{P_1 l_1}{AE} - \frac{P_2 l_2}{AE} + \frac{P_3 l_3}{AE} = \frac{1}{AE}(P_1 l_1 - P_2 l_2 + P_3 l_3)$$

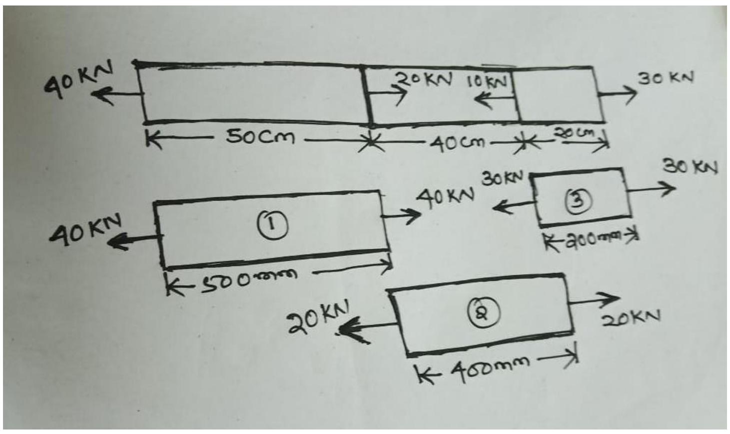
$$\delta l = \frac{1}{AE}(P_1 l_1 - P_2 l_2 + P_3 l_3) = \frac{(100 \times 10^3 \times 500\ - 100 \times 10^3 \times 1000 + 100 \times 10^3 \times 500)}{490.87 \times 2 \times 10^5} = 0$$

$$\therefore \boldsymbol{deformation\ of\ bar\ \delta l = 0}$$

Q.5 A steel bar 25mm. diameter is loaded as shown in figure. Determine the stress in each part and the total elongation. E = 2 x 10^5 N/mm^2 [2014wn]

Ans.

Given Data:



Diameter of bar d = 25 mm

Young's Modulus $E = 2 \times 10^5 MPa = 2 \times 10^5 \ {}^{N}/_{mm^2}$

$Cross - Sectional\ Area\ of\ Bar$

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 25^2 = 490.87\ mm^2$$

Splitting of forces and body into three Parts

Now we have

$P_1 = 40 \times 10^3\ N\ (Pull), \qquad l_1 = 500\ mm,$

$P_2 = 20 \times 10^3\ N (pull), \qquad l_2 = 400\ mm$

$P_3 = 30 \times 10^3\ N (pull), \qquad l_3 = 200\ mm$

Applying principle of Super Position theorem

$$Total\ deformation\ of\ Bar\ \delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \frac{P_1 l_1}{AE} - \frac{P_2 l_2}{AE} + \frac{P_3 l_3}{AE} = \frac{1}{AE}(P_1 l_1 - P_2 l_2 + P_3 l_3)$$

$$\delta l = \frac{1}{AE}(P_1 l_1 - P_2 l_2 + P_3 l_3) = \frac{(40 \times 10^3 \times 500 + 20 \times 10^3 \times 400 + 30 \times 10^3 \times 200)}{490.87 \times 2 \times 10^5}$$

$$\boldsymbol{\delta l = 0.3462\ mm}$$

$$\therefore\ \boldsymbol{Total\ deformation\ of\ Bar\ \delta l = 0.3462\ mm}$$

Stresses in Each Part:

$$Stress\ in\ part\ 1, \quad \sigma_1 = \frac{P_1}{A} = \frac{40 \times 10^3}{490.87} = 81.46\ N/mm^2$$

$$Stress\ in\ part\ 2, \quad \sigma_2 = \frac{P_2}{A} = \frac{20 \times 10^3}{490.87} = 40.73\ N/mm^2$$

$$Stress\ in\ part\ 3, \quad \sigma_3 = \frac{P_3}{A} = \frac{30 \times 10^3}{490.87} = 61.09\ N/mm^2$$

Stresses in Composite Bar:

Composite Bar:

It is a bar made up of two or more different materials joined together in such a way that the system extends or contracts one unit equally when subjected of tension or compression.

While solving problems on composite Bar

i. Extension and contraction of bar being equal strain i.e. the deformation/unit length of bar is also equal.
ii. Total external load on the bar is equal to the algebraic sum of Loads carried different materials.

Let's consider a composite bar as shown in diagram

P - load applied on composite Bar

P_1 – load applied in Material '1'

P_2 – Load applied in Material '2'

δl_1 – deformation of Material '1'

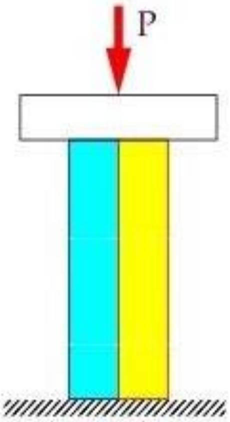
δl_2 – deformation of Material '2'

E_1 – Young' s Modulus for Material '1'

E_2 – Young' s Modulus for Material '2'

σ_1 - stress induced in Material '1'

σ_2 - stress induced in Material '2'



$$P_1 + P_2 = P$$

$$\delta L_1 = \delta L_2 = \delta L$$

$$\frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2}$$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \Rightarrow \sigma_1 = \boxed{\frac{E_1}{E_2}}\sigma_2$$

Modular ratio

Q. A reinforced concrete column is 300 mm x 300 mm in section. The column is provided with 8 bars of 20 mm dia. The column carries a load of 360 KN. Find the stresses in concrete and the steel bars. Take Es = 2.1 x 10^5 N/mm^2 and E$_c$= 0.14 x 10^5 n/mm^2. [2011(W), 2012, 2015, 2016]

Ans.:

Given Data:

$Rectangular\ Column = 300\ mm\ \times 300\ mm$

$No\ of\ steel\ Bar = 08$

$Diameter\ of\ Steel\ bar\ d_S = 20\ mm$

$Applied\ Load\ P = 360\ KN = 360 \times 10^3\ N$

$E_S = 2.1 \times 10^5\ {N}/{mm^2}$

$E_C = 0.14 \times 10^5\ {N}/{mm^2}$

$\sigma_S, \sigma_C = ?$

$Area\ of\ steel\ bars,\quad A_S = 8 \times \frac{\pi}{4} \times d_S^{\ 2} = 8 \times \frac{\pi}{4} \times 20^2 = 2513.27\ mm^2$

$Area\ of\ rectangular\ Column\ A_{Column} = 300 \times 300 = 90000 mm^2$

$Area\ of\ Concrete\ A_c = A_{Column} - A_S = 90000 - 2513.27 = 87486.72\ mm^2$

We have in case of Composite bar

a. $Strain\ in\ Concrete = strain\ in\ Steel$

$$\Rightarrow e_c = e_s \Rightarrow \frac{\sigma_C}{E_C} = \frac{\sigma_S}{E_S} \Rightarrow \frac{E_S}{E_C} \times \sigma_C = \sigma_S$$

$$\Rightarrow \frac{2.1 \times 10^5}{0.14 \times 10^5} \times \sigma_C = \sigma_S$$

$$\Rightarrow \mathbf{15\sigma_C = \sigma_S} \ \dots\dots\dots\dots (i)$$

b. $Total\ load\ applied\ P = P_C + P_S \Rightarrow P = \sigma_S . A_S + \sigma_C . A_c = 360 \times 10^3$

$$\Rightarrow 360 \times 10^3 = 15\sigma_C . A_S + \sigma_C . A_c = \sigma_C(15A_S + A_c)$$

$$\Rightarrow 360 \times 10^3 = \sigma_C(15 \times 2513.27 + 87486.72)$$

$$\Rightarrow \frac{360 \times 10^3}{(15 \times 2513.27 + 87486.72)} = \sigma_C, \qquad \Rightarrow \boldsymbol{\sigma_C = 2.87\ {N}/{mm^2}}$$

Now stress in steel bar $\Rightarrow 15\sigma_C = \sigma_S \quad \dots\dots\dots\dots (i) \qquad \Rightarrow \boldsymbol{\sigma_S = 43.05\ {N}/{mm^2}}$

Q. A reinforced concrete circular section of $5 \times 10^4 mm^2$, cross-sectional area carries 6 reinforcing bars whose total area is 500 mm^2. Find the safe load, the column can carry, if the concrete is not to be stressed more than 3.5 MPa .Take modular ratio for steel and concrete as 18.

Ans.:

Given Data:

$Area\ of\ circular\ Column\ A_{Column} = 5 \times 10^4 mm^2$

$Area\ of\ steel\ bars,\quad A_S = 500\ mm^2$

$No\ of\ steel\ Bar = 06$

$\frac{E_S}{E_C} = 18$

$P = ?$

$\sigma_C = 3.5\ MPa = 3.5\ \boldsymbol{{N}/{mm^2}}$

$Area\ of\ Concrete\ A_c = A_{Column} - A_S = 50000 - 500 = 49500\ mm^2$

We have in case of Composite bar

a. $Strain\ in\ Concrete = strain\ in\ Steel$

$$\Rightarrow e_c = e_s \quad \Rightarrow \frac{\sigma_C}{E_C} = \frac{\sigma_S}{E_S} \quad \Rightarrow \frac{E_S}{E_C} \times \sigma_C = \sigma_S$$

$$\Rightarrow 18\sigma_C = \sigma_S \ \Rightarrow 18 \times 3.5 = 63 = \boldsymbol{\sigma_S}$$

$$\boldsymbol{\sigma_S} = 63\ {N}/{mm^2} \ \dots\dots\dots\dots (i)$$

b. $Total\ load\ applied\ \ on\ column$

$$P = P_C + P_S \Rightarrow P = \sigma_S . A_S + \sigma_C . A_c$$

$$\Rightarrow P = 63 \times 500 + 3.5 \times 49500$$
$$\Rightarrow \boldsymbol{P = 204750\ N\ say\ 204.75\ KN}$$

Q. A reinforced concrete column 500 mm × 500 mm in section is reinforced with 4 steel bar of 30 mm diameter. One in each corner .the column is carrying a load 1000KN. find the stresses in concrete and steel bar, take E for steel 210 G Pa and for Concrete 14 G Pa

Ans.:-

Given data:

Size of column- 500×500 mm

No of steel bar (n) – 4

Dia. of steel Rod $d_s = 30$ mm

$P = 1000\times10^3$ N

$E_s = 210$ G Pa $= 210\times10^3$ N/mm^2

$E_c = 14$ G Pa $= 14\times10^3$ N/mm^2

In case of composite bar

Area of column $A_{column} = 500\times500 = 250000$ mm^2

Total steel area $A_s = n \times \frac{\pi}{4} \times d_s^{\ 2}$

$A_s = 4 \times \frac{\pi}{4} \times 30^2$ mm^2 = 2827.43 mm^2

Now area of concrete $(A_c) = A_{column} - A_s = 250000-2817.43= 247172.57$ mm^2

In case of composite bar

Total Strain in steel = Total strain in concrete

$e_s = e_c$

$$\Rightarrow \frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\Rightarrow \sigma_s = \frac{E_s}{E_c} \times \sigma_c$$

$$\Rightarrow \sigma_s = \frac{210}{14} \times \sigma_c$$

$$\Rightarrow \sigma_s = 15.\sigma_c \cdots\cdots\cdots (i)$$

Total load acting on bar

$P = P_s + P_c = \sigma_s \times A_s + \sigma_c \times A_c = 15.\sigma_c \times A_s + \sigma_c \times A_c$ .... (ii)

$\Rightarrow 1000*10^3 = \sigma_c(15A_s + A_c)$ ($\therefore$putting value of $\sigma_s = 15.\sigma_c$ $from\ equaion - i$

$$\Rightarrow \sigma_c = \frac{1000 \times 10^3}{(15 \times 2827.43 + 247172.57)}$$

$\Rightarrow$ $\boldsymbol{\sigma_c}$**= 3.45 M Pa** → stress induced in concrete

$\boldsymbol{\sigma_s} = 15.\sigma_c = 15 \times 3.45 = \boldsymbol{51.75\ MPa}$ → Stress induced in steel

➢ Temperature Stress:

When the temperature of a material changes there will be a corresponding change in its dimensions. When the member is free to expand or contract due to rise and fall of temperature , no stress is induced in member, But if the natural change in length due to rise and fall of temp. be prevented stress will be induced ,which is called temperature stress .

Temperature stress is also known as compressive stress whenever the ends of member are fixed and is subjected to rise and fall of temperature while its natural expansion is prevented and Compressive stress is induced in it.

$let's consider\ a\ bar\ AB\ \ whose\ on\ end\ is\ fixed\ at\ A$

$l - length\ of\ bar,$

$\Delta T - Change\ in\ Temperature$

$\alpha - Coefficient\ of\ Linear\ expansion$

$due\ to\ rise\ or\ fall\ of\ Tempearature\ applied\ on\ it\ then\ the\ free\ expansion\ of\ bar\ BB'$

$$\delta l = l.\alpha.\Delta T$$

If the bar is subjected to external load 'P' at its end B, then the length of rod is decreased to length 'l'

$$Strain\ induced\ in\ bar\ e = \frac{\delta l}{l} = \frac{l.\alpha.\Delta T}{l} = \alpha.\Delta T$$

$$\boldsymbol{e = \alpha.\Delta T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. Temp.strain\ in\ Bar$$

$if\ A - area\ of\ cross - section\ of\ bar\ now\ stress\ induced\ bar\ due\ to\ external\ load\ P$

$$\sigma = \frac{P}{A}$$

$$We\ have\ young's\ Modulus\ of\ Elasticity\ E = \frac{\sigma}{e} \qquad \Rightarrow \sigma = E.e = E.\alpha.\Delta T$$

$$\Rightarrow \boldsymbol{\sigma = E.\alpha.\Delta T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. Temp.Stress\ induced\ in\ Bar$$

Q. What do you mean by temperature stress? Write its Formula. [2017, 2016, 2014, 2013, 2012, 2010]

Ans.: When the temperature of a material changes there will be a corresponding change in its dimensions. When the member is free to expand or contract due to rise and fall of temperature , no stress is induced in member, But if the natural change in length due to rise and fall of temp. be prevented stress will be induced ,which is called temperature stress .

$$\boldsymbol{\sigma = E.\alpha.\Delta T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. Temp.Stress\ induced\ in\ Bar$$

$\Delta T - Change\ in\ Temperature$

$\alpha - Coefficient\ of\ Linear\ expansion$

$E - young's\ Modulus\ of\ Elasticity$

Q. **A steel rod 25mm in diameter and 4m long is connected to two grips at each end at a temp. of 120°C. Find the Pull exerted when temp. Falls to 35°C take E = 2×10^5N/mm^2,** $\alpha_s = 1.2 \times 10^{-5}$ /°C

Ans.:

Given Data:

$d = 25\ mm$

$l = 4m = 4000mm$

$T_1 = 120°C$

$T_2 = 35°C$

$\alpha_S = 1.2 \times 10^{-5}/°C$

E = 2×10^5N/mm^2

The change in temp $\Delta T = T_1 - T_2 = 60°C - 20°C = 40°C$

Area of steel Rod $A_s = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 25^2 = 490.87\ mm^2$

Natural change in length

$$\delta l = l.\alpha.\Delta T = 4000 \times 0.000012 \times 85 = 4.8\ mm$$

$we\ have\ Temp.Stress\ induced\ in\ railway\ ine$

$$\sigma = E.\alpha.\Delta T$$

$$\Rightarrow \frac{P}{A_s} = E.\alpha.\Delta T$$

$\Rightarrow P = E.\alpha.\Delta T.A_s = 2 \times 10^5 \times 0.000012 \times 85 \times 490.87 = 100137.48\ Nsay\ 100.137\ KN$

$\therefore$ Pull exerted on steel rod, $\boldsymbol{P = 100.137\ KN}$

Q. A steel rod 25 mm dia. & 4000mm long is subjected to tensile force P acting axially. The temp. of rod is then raised through 85°C and total extension measured as 0.35mm. Calculate the value of 'P' take E_s = 200GN/m^2 and $\alpha_s = 12 \times 10^{-6}$ / °C.

Ans:

Given Data:

$d = 25\ mm$
$l = 4m$
$\Delta T = 85°C$
$E = 2 \times 10^5 N/mm2$
$\alpha_s = 1.2 \times 10^{-5}/°C$
$\boldsymbol{P} = ?$

$$Area\ steel\ rod, A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 25^2 = 490.87\ mm^2$$

$$Due\ to\ tensile\ load\ P, \quad \delta l_1 = \frac{Pl}{AE}$$

$$due\ to\ rise\ in\ temperature\ in\ bar\ \delta l_2 = l.\alpha_S.\Delta T$$

$Now\ we\ have\ total\ deformation\ of\ bar$

$$\delta l = \delta l_1 + \delta l_2$$

$$\Rightarrow \delta l = \frac{Pl}{AE} + l.\alpha_S.\Delta T = 0.35$$

$$\Rightarrow Pl = 0.35 \times A \times E - l.\alpha_S.\Delta T$$

$$\Rightarrow P = \frac{0.35 \times A \times E - l.\alpha_S.\Delta T}{l}$$

$$\Rightarrow P = \frac{0.35 \times 490.87 \times 2 \times 10^5 - 4000 \times 1.2 \times 10^{-5} \times 85}{4000} = 8590.223N\ say\ 8.59\ KN$$

$$\therefore\ Tensile\ load\ acting\ on\ steel\ bar\ \boldsymbol{P} = \mathbf{8.59}\ \boldsymbol{KN}$$

Q. Find expression for temperature stress in case of a bar of length 1 m fixed at both ends subjected to heating through t° C.(I) If the ends do not yield. (ii) If the ends yield by δ. Take α as the linear coefficient of expansion & E as the young's modulus.

Ans.:

$let's$ consider a bar ahving
$l = length\ of\ bar$
$E - young's\ modulus\ for\ material\ of\ bar$
$e - temp\ strain\ in\ bar$
$\sigma - stress\ induced\ in\ bar$
$\delta l - change\ in\ length\ of\ bar$
$\Delta T - change\ in\ Temp\ of\ bar$

i. Temperature stress induced in bar when ends are not yield:
Whenever the two ends of bar are not yield then there will be natural expansion in length of bar takes place due to change in temp.

Now the natural expansion in length of bar

$$\delta l = l.\alpha.\Delta T$$

$$Strain\ induced\ in\ bar\ e = \frac{\delta l}{l} = \frac{l.\alpha.\Delta T}{l} = \alpha.\Delta T$$

$$We\ have\ young's\ Modulus\ of\ Elasticity\ E = \frac{\sigma}{e} \quad \Rightarrow \sigma = E.e = E.\alpha.\Delta T$$

$$\Rightarrow \boldsymbol{\sigma = E.\alpha.\Delta T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .\ Temp.Stress\ induced\ in\ Bar$$

ii. Temperature stress induced in bar when ends are yield by deformation $'\delta'$

Whenever the two ends of bar are yield by deformation $'\delta'$ then there will be expansion in length of bar permitted only by $'\delta'$,
Now the actual expansion in length of bar

$$\delta l = l.\alpha.\Delta T - \delta$$

$$Strain\ induced\ in\ bar\ e = \frac{\delta l}{l} = \frac{l.\alpha.\Delta T - \delta}{l}$$

$$We\ have\ young's\ Modulus\ of\ Elasticity\ E = \frac{\sigma}{e} \quad \Rightarrow \sigma = E.e = E.\frac{l.\alpha.\Delta T - \delta}{l}$$

$$\Rightarrow \boldsymbol{\sigma = E.\frac{l.\alpha.\Delta T - \delta}{l}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.Temp.Stress\ induced\ in\ Bar$$

Q. A railway line is laid so that there is no stress in the rails at 60 ℃. Calculate stress in the rail at 20 ℃ if

I. **No allowance is made for contraction.**
II. **There is allowance of 0.005 mm for contraction per rail.**

The rails are 30 mm long , $E = 210\ GPa, \alpha = 0.000012/$℃

Ans.:

Given Data:

$T_1 = 60℃$

$T_2 = 20℃$

$l = 30mm$

$E = 200\ GPa = 200 \times 10^3\ {}^{N}/_{mm^2}$

$\alpha = 0.000012/℃$

i. Temp stress when there is no allowance for contraction

The change in temp $\Delta T = T_1 - T_2 = 60℃ - 20℃ = 40℃$

Natural change in length

$$\delta l = l.\alpha.\Delta T = 30 \times 0.000012 \times 40 = 0.0144\ mm$$

$$Strain\ induced\ in\ ailway\ line\ , \quad e = \frac{\delta l}{l} = \frac{l.\alpha.\Delta T}{l} = \alpha.\Delta T = 0.000012 \times 40 = 0.00048$$

$Temp.Stress\ induced\ in\ railway\ ine$

$$\boldsymbol{\sigma = E.\alpha.\Delta T}$$

$\boldsymbol{\sigma} = 200 \times 10^3 \times 0.000012 \times 40 = 96\ {}^{N}/_{mm^2}$

$$\boldsymbol{\sigma = 96\ {}^{N}/_{mm^2}}$$

ii. Temp stress when there is allowance of 5mm

Actual expansion in length of bar

$$\delta l = l.\alpha.\Delta T - \delta = 30 \times 0.000012 \times 40 - 0.005 = 9.4 \times 10^{-3}$$

$$Strain\ induced\ in\ bar\ e = \frac{\delta l}{l} = \frac{l.\alpha.\Delta T - \delta}{l} = \frac{9.4 \times 10^{-3}}{30} = 3.133 \times 10^{-4}$$

$Temp.Stress\ induced\ in\ railway\ ine$

$$\sigma = E.e = 200 \times 10^3 \times 3.133 \times 10^{-4} = 65.8\ {}^{N}/_{mm^2}$$

$$\boldsymbol{\sigma = 65.8\ {}^{N}/_{mm^2}}$$

- Temperature Stress in Composite Bar: Let's consider a composite bar consisting of two or more different materials. When subject to change in temperature, it causes the bar to expand or Contract on basis of different coefficient of linear expansion of the two material do not expand or contract by same amount but expand or contract by different amount as shown in fig.

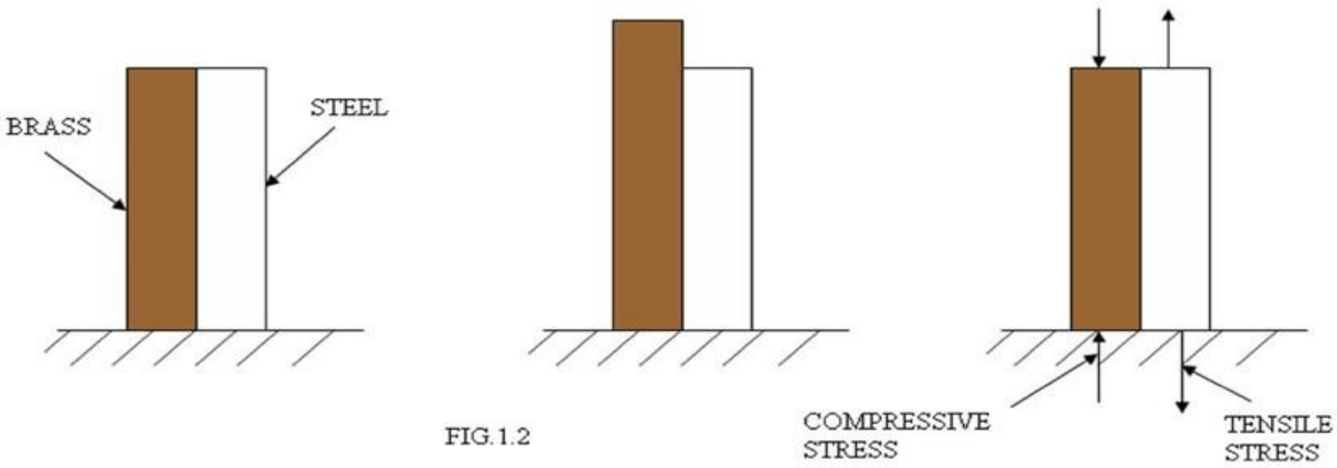


FIG.1.2

Now consider a composite bar consists of two materials as Steel and Brass as shown by above fig.
Let's the bar is heated through some temperature, due to which there will be an expansion of two members takes place. As they are rigidly fixed with each other, there will be same expansion between two materials.

We know that the coefficient of linear expansion of Brass is more than steel so therefore the free expansion of brass will be more than steel. But since both members are not free to expand ,therefore the expansion of composite bar as a whole will be less than brass , to keep them on a single core a tensile stress is induced in Steel member and Compressive stress is induced in Brass member is observed

In case of problems solved for composite bar Subjected to Rise and Fall of Temperature

i. Compressive Load applied on Brass = Tensile load applied on Steel

i.e. $P_B = P_S$

$$\Rightarrow \sigma_B . A_B = \sigma_S . A_S \quad \Rightarrow \boldsymbol{\sigma_S} = \frac{\boldsymbol{\sigma_B . A_B}}{\boldsymbol{A_S}}$$

$Where$
$P_B - Compressive\ load\ applied\ on\ Brass$
$P_S - Tensile\ load\ on\ Steel\ Load$
$\sigma_S - Tensile\ stress\ induced\ in\ steel$
$\sigma_B - Compressive\ stress\ induced\ in\ Brass$
$A_S - Area\ of\ Crosssection\ of\ Steel\ Bar$
$A_B - Areaof\ Crosssection\ of\ Brass\ Bar$

ii. Actual Deformation of Brass bar = Actual Deformation Steel Bar

i.e. $\delta l_B = \delta l_S$

$$\Rightarrow l_B . \alpha_B . \Delta T - \delta l = l_S . \alpha_S . \Delta T + \delta l$$

$$\Rightarrow l_B . \alpha_B . \Delta T - \frac{P_B . l}{A_B . E_B} = l_S . \alpha_S . \Delta T + \frac{P_S . l}{A_S . E_S}$$

$$\Rightarrow \alpha_B.\Delta T - \frac{\sigma_B}{E_B} = \alpha_S.\Delta T + \frac{\sigma_S}{E_S} \ldots\ldots\ldots\ldots\ldots..\textit{deviding length of bar in each equation}$$

$$\Rightarrow \alpha_B.\Delta T - \alpha_S.\Delta T = \frac{\sigma_B}{E_B} + \frac{\sigma_S}{E_S}$$

$$\Rightarrow \frac{\boldsymbol{\sigma_B}}{\boldsymbol{E_B}} + \frac{\boldsymbol{\sigma_S}}{\boldsymbol{E_S}} = (\boldsymbol{\alpha_B - \alpha_S})\boldsymbol{\Delta T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\textbf{Total deformation in composite bar}$$

Q. A copper rod 6cm in diameter is placed with in a steel tube of external diameter 8 cm and internal dia. 6cm exactly of same length. Calculate Stresses in copper and steel tube. If temp raised to a 50^0 C?
Es =210GPa, α_s = 11.5 x 10^{-6} / ºC Ecu = 105GPa, α_{cu} = 17 x 10^{-6} / ºC

Ans.

Data Given:

$d_{cu} = 6\ cm = 60\ mm$

$d_{s_e} = 8cm = 80\ mm$

$d_{s_i} = 6cm = 60mm\ \Delta T - 50^{\circ}C$

$E_S = 210\text{GPa} = 210 \times 10^3\ {}^{N}/_{mm^2}$

$E_{Cu} = 105GPa = 105 \times 10^3\ {}^{N}/_{mm^2}$

$\alpha_S = 11.5 \times 10^{-6}$ /oC

$\alpha_{cu} = 17 \times 10^{-6}$ /oC

Area of copper tube, $A_{cu} = \frac{\pi}{4} \times d_{cu}^2 = \frac{\pi}{4} \times 60^2 = 2827.43\ mm^2$

Area of steel tube, $A_s = \frac{\pi}{4} \times (d_{s_e}^2 - d_{s_i}^2) = \frac{\pi}{4} \times (80^2 - 60^2) = 2199.11\ mm^2$

1. Compressive Load applied on Copper = Tensile load applied on Steel

i.e. $P_{cu} = P_S$

$$\Rightarrow \sigma_{cu}.A_{cu} = \sigma_S.A_S$$

$$\Rightarrow \sigma_S = \sigma_{cu}\left(\frac{A_{cu}}{A_S}\right) = \sigma_{cu}\left(\frac{2827.43}{2199.11}\right)$$

$$\Rightarrow \boldsymbol{\sigma_S = 1.28\ \sigma_{cu}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(\boldsymbol{i})$$

2. Actual Deformation of copper Tube = Actual Deformation Steel Tube

$$\frac{\sigma_{cu}}{E_{Cu}} + \frac{\sigma_S}{E_S} = (\alpha_{cu} - \alpha_S)\Delta T$$

$$\Rightarrow \boldsymbol{\sigma_{cu}}\left(\frac{1}{E_{Cu}} + \frac{1.28}{E_S}\right) = (\alpha_{cu} - \alpha_S)\Delta T$$

$$\Rightarrow \left(\frac{E_S + 1.28.E_{Cu}}{E_{Cu}.E_S}\right)\boldsymbol{\sigma_{cu}} = (\alpha_{cu} - \alpha_S)\Delta T$$

$$\Rightarrow \boldsymbol{\sigma_{cu}} = \frac{(\alpha_{cu} - \alpha_S)\Delta T}{\left(\frac{E_S + 1.28.E_{Cu}}{E_{Cu}.E_S}\right)} = \frac{(17 \times 10^{-6} - 11.5 \times 10^{-6}) \times 50}{\left(\frac{210 \times 10^3 + 1.28 \times 105 \times 10^3}{105 \times 10^3 \times 210 \times 10^3}\right)}$$

$$\Rightarrow \boldsymbol{\sigma_{cu}} = \frac{(17 \times 10^{-6} - 11.5 \times 10^{-6}) \times 50}{0.000015619} = 17.60\ {}^{N}/_{mm^2}$$

$\therefore$ *Stress in copper tube* $\boldsymbol{\sigma_{cu} = 17.60\ {}^{N}/_{mm^2}}$

now $\therefore$ *stress in steel tube* $\boldsymbol{\sigma_S = 1.28} \times 17.60\ {}^{N}/_{mm^2} = \mathbf{22.52\ {}^{N}/_{mm^2}}$

Q. A **15 mm dia. Steel rod passes centrally through a copper tube 50 mm external diameter and 40 mm internal diameter . the tube is closed at each end by rigid plate of negligible thickness. The nuts are tightened lightly home on the projecting parts of rod if the temp. of assembly is raised by 60℃ . calculate the stresses developed in steel and copper. (2015 W bp)**

$\boldsymbol{E_S = 200\text{GPa} = 200 \times 10^3\ N/_{mm^2}}$ $\boldsymbol{E_{Cu} = 105GPa = 105 \times 10^3\ N/_{mm^2}}$ $\boldsymbol{\alpha_S = 12 \times 10^{-6}}$ /℃

$\boldsymbol{\alpha_{cu} = 17.5 \times 10^{-6}}$ /℃

Dia. of steel (d_s) = 15mm

Dia. of cupper (d_{ci}) = 40 mm

(d_{ce}) = 50 mm

Rise in temperature $\Delta T = 60^0$C

$E_S = 200\text{GPa} = 200 \times 10^3\ N/_{mm^2}$

$E_{Cu} = 105GPa = 105 \times 10^3\ N/_{mm^2}$

$\alpha_S = 12 \times 10^{-6}$ /℃

$\alpha_{cu} = 17.5 \times 10^{-6}$ /℃

For composite bar

Area of steel $A_s = \frac{\pi}{4} \times d_s^2 = \frac{\pi}{4} * 15^2 = 176.71\ \text{mm}^2$

Area of cupper $A_c = \frac{\pi}{4} \times (d_{ce}^2 - d_{ci}^2) = \frac{\pi}{4} * (50^2 - 40^2) = 706.85\ \text{mm}^2$

As the natural expansion of cupper is more than that of steel

The push in cupper = pull on steel

$P_s = P_c$

$\Rightarrow \sigma_s \times A_s = \sigma_c \times A_c$

$\Rightarrow \sigma_s = \frac{A_c}{A_s} \times \sigma_c = \frac{706.85}{176.71} = 4\sigma_c$

$\Rightarrow \sigma_s = 4\sigma_c \cdots\cdots\cdots\cdots\cdots (i)$

Now total strain in composite bar

$e_c + e_s = (\alpha_c - \alpha_s)\Delta T$

$\frac{\sigma_s}{E_s} + \frac{\sigma_c}{E_c} = (\alpha_c - \alpha_s)\Delta T \Rightarrow \sigma_c\left(\frac{4}{E_s} + \frac{1}{E_c}\right) = (\alpha_c - \alpha_s)\Delta T \qquad (\because\ \sigma_s = 4\sigma_c \cdots\cdots\cdots\cdots\cdots (i))$

$\Rightarrow \sigma_c\left(\frac{4}{E_s} + \frac{1}{E_c}\right) = (\alpha_c - \alpha_s)\Delta T$

$\Rightarrow \sigma_c\left(\frac{4}{E_s} + \frac{1}{E_c}\right) = (\alpha_c - \alpha_s)\Delta T$

$\Rightarrow \sigma_c\left(\frac{4 \times E_c + E_s}{E_s \times E_c}\right) = (\alpha_c - \alpha_s)\Delta T$

$\Rightarrow \sigma_c = \frac{E_s \times E_c}{4 \times E_c + E_s} \times (\alpha_c - \alpha_s)\Delta T = \frac{2 \times 10^5 \times 1 \times 10^5}{4 \times 1 \times 10^5 + 2 \times 10^5} \times (17 - 11) \times 60 = 12\ \frac{N}{mm^2}$

$\boldsymbol{\Rightarrow \sigma_c = 12\ \frac{N}{mm^2}\ or\ 12\ MPa \rightarrow stress\ in\ cupper}$

Now from equation -i

We have

$\sigma_s = 4\sigma_c \cdots\cdots\cdots\cdots\cdots (i)$),

$\boldsymbol{\sigma_s = 4\sigma_c = 4 \times 12 = 48\ MPa\ \rightarrow stress\ acting\ in\ steel}$

➢ Strain energy:

Definition: it is the energy stored or absorbed by a member when work is done on it to deform it.

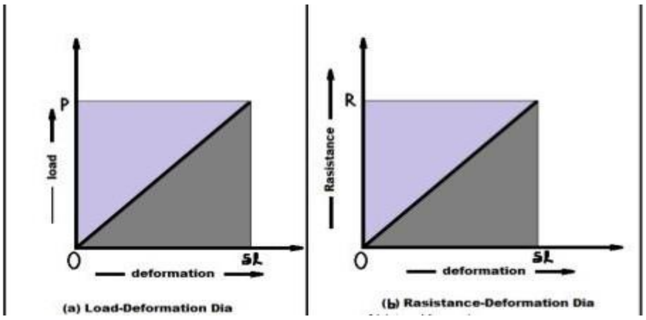
It is also the energy stored in body up to the elastic limit

It is denoted by symbol 'U'

Expression for Strain energy:

Let's consider a bar having

l- Length of bar A- Area of Cross section of bar



(a) Load-Deformation Dia (b) Rasistance-Deformation Dia

It is fixed at its upper end is subjected to load 'P'. Under action of tensile load on bar it deforms along longitudinally as $'\delta l'$

If we plot a graph between load and deformation, then from the graph area under the triangle gives the energy stored in member. The area under the curve is also known as the Work done by load on body

Now $Strain\ energy\ stored\ in\ member\ =\ Work\ done\ against\ resistance\ of\ the\ bar$

$$U = avarage\ Resistance\ \times displacement = \frac{R}{2} \times \delta l$$

$$U = \frac{P}{2} \times \delta l \ldots\ldots\ldots\ldots\ldots Since\ \ load = resistance$$

$$U = \frac{\sigma . A}{2} \times \frac{\sigma}{E} \times l\ = \frac{\sigma^2}{2E} Al$$

$$\boldsymbol{U = \frac{\sigma^2}{2E} Al} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . Strain\ energy\ stored\ in\ Body$$

$$where\ \sigma - Stress\ induced\ in\ bar, \qquad E - Youngs\ modulus\ of\ elasticity\ for\ material\ of\ body$$

➢ Resilience:

it is defined as the energy stored inside the body due to its deformation under action of load, within in elastic limit. It is similar to the strain energy induced in body.

➢ Modulus of Resilience:

It is defined as the strain energy stored per unit volume of the material when it is stressed to the proportional limit is called Modulus of resilience. $Modulus\ of\ Resilience = \boldsymbol{\frac{\sigma^2}{2E}}$

Q. What is strain energy and resilience? [2017W, 2015, 2011]

Strain energy: It is the energy stored in body during its deformation under the action of external load on it. Up to the elastic limit

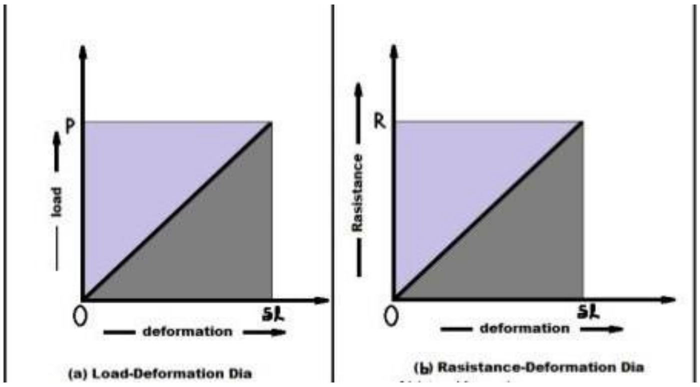
Resilience**:** it is defined as the energy stored inside the body due to its deformation under action of load, within in elastic limit. It is similar to the strain energy induced in body

➢ Determination of Stresses induced in body

i. Gradually applied Load
ii. Suddenly applied Load
iii. Impact Load

- Stress due to Gradually applied Load:

In case of gradually applied load on the body, the magnitude of load gradually increases from Zero to maximum value and there after remains constant and average value of load is considered while calculating Strain energy.
Now



$$Strain\ energy\ stored\ in\ member\ =\ Work\ done\ against\ resistance\ of\ the\ bar$$

$$\Rightarrow U = avarage\ Resistance\ \times displacement = \frac{R}{2} \times \delta l = \frac{P}{2} \times \delta l$$

$$\Rightarrow \frac{\sigma^2}{2E} Al = \frac{P}{2} \times \frac{\sigma}{E} \times l$$

$$\Rightarrow \boldsymbol{\sigma} = \frac{\boldsymbol{P}}{\boldsymbol{A}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots stress\ due\ to\ Gradual\ applied\ load$$

- Stress due to Suddenly applied Load:

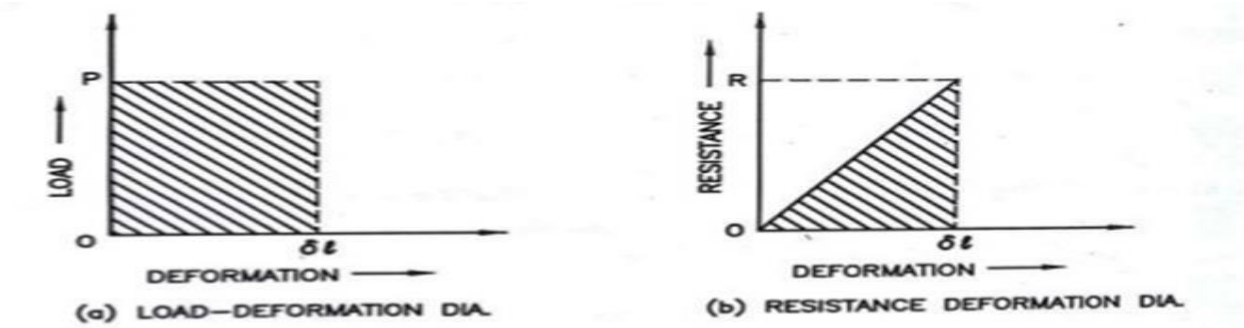
In case of suddenly applied load the magnitude of load ‘P’ is constant throughout process of extension of bar

$$Strain\ energy\ stored\ in\ member\ =\ Work\ done\ against\ resistance\ of\ the\ bar$$

$$\Rightarrow U = Resistance\ \ \times displacement = R \times \delta l = P \times \delta l$$

$$\Rightarrow \frac{\sigma^2}{2E} Al = P \times \frac{\sigma}{E} \times l$$

$$\Rightarrow \boldsymbol{\sigma} = 2\frac{\boldsymbol{P}}{\boldsymbol{A}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots stress\ due\ to\ Suddenly\ \ applied\ load$$



Q. What is the ratio of stress when the body is loaded gradually and to when loaded suddenly?

[2018 S]

During Suddenly applied load the stress induced in bar $\sigma = 2\ \frac{P}{A}$

During Gradual applied load the stress induced in bar $\sigma = \ \frac{P}{A}$

Now ratio of stresses due to both load $\ {2\ \frac{P}{A}}/{\frac{P}{A}} = 2$

$$i.e. stress\ in\ sudden\ applied\ load\ \ is\ two\ times\ of\ stress\ in\ gradual\ loading$$

- Stress induced during Impact Load:

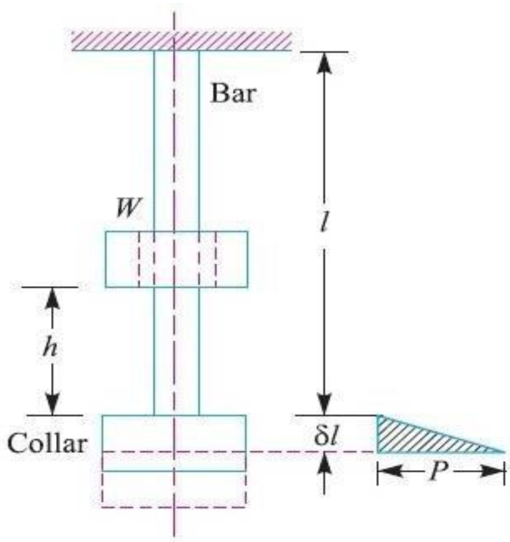
Impact Load: Whenever the load is acting on the body with a initial velocity, then the load is called Impact load.

Very often structural components are subjected to impact or dynamic load, which produces stresses much higher than what are stresses much higher than what are caused by same load applied gradually.

In case of impact load stress is determined by principle of conservation of energy. The potential energy of Nut is causing the deformation of collar fixed with lower end of bar.

Let's consider a bar whose one end is fixed and other end is attached with a Circular Collar a shown in figure.

A nut is falling through some height striking upon the collar causing deformation bar.



$let\ onsider$
$A - area\ of\ crosssection\ of\ bar$
$h - height\ through\ Nut\ is\ falling\ upon\ Collar$
$l - length\ of\ bar$
$\sigma_{instant} - nstantaneous\ stress\ induced\ in\ bar$
$\delta l - change\ in\ length\ of\ bar$
$U - strain\ energy\ stored\ in\ bar$
$E - young's modulus\ of\ elasticity\ for\ material\ of\ bar$
$W - weight\ of\ Nut$

$$Strain\ energy\ stored\ in\ member\ = Work\ done\ against\ resistance\ of\ the\ bar$$

$$\Rightarrow U = Resistance \times displacement = P(h + \delta l)$$
$$\Rightarrow U = W(h + \delta l)$$

$$\Rightarrow \frac{\sigma^2}{2E} Al = W\left(h + \frac{\sigma}{E} \times l\right) = Wh + W\frac{\sigma}{E} \times l$$
$$\Rightarrow \frac{\sigma^2}{2E} Al = Wh + W\frac{\sigma}{E} \times l$$

$$\Rightarrow \frac{\sigma^2}{2E} Al - Wh - W\frac{\sigma}{E} \times l = 0$$
$$\Rightarrow \frac{\sigma^2}{2} - \frac{EWh}{AL} - \sigma\left(\frac{W}{A}\right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .\ deviding\ \frac{E}{AL}\ in\ all\ the\ terms$$
$$\Rightarrow \frac{\sigma^2}{2} - \sigma\frac{E}{L}.h - \sigma\left(\frac{W}{A}\right) = 0$$

Solving the above Quadratic Equation

$$\boldsymbol{\sigma_{instant}} = \frac{-\left(-\frac{W}{A}\right) \pm \sqrt{\left(-\frac{W}{A}\right)^2 - 4.\frac{1}{2}\left(-\frac{EWh}{Al}\right)}}{2 \times \frac{1}{2}}$$

$$\boldsymbol{\sigma_{instant}} = \left(\frac{W}{A}\right) \pm \sqrt{\left(\frac{W}{A}\right)^2 + \left(\frac{2EWh}{Al}\right)}$$

Now maximum instantaneous stress $\boldsymbol{\sigma_{Maximum}} = \left(\frac{\boldsymbol{W}}{\boldsymbol{A}}\right) + \sqrt{\left(\frac{\boldsymbol{W}}{\boldsymbol{A}}\right)^{\boldsymbol{2}} + \left(\frac{\boldsymbol{2EWh}}{\boldsymbol{Al}}\right)}$

Now maximum instantaneous stress , $\boldsymbol{\sigma_{Minimum}} = \left(\frac{W}{A}\right) - \sqrt{\left(\frac{W}{A}\right)^2 + \left(\frac{2EWh}{Al}\right)}$

Q. **A cupper bar 1.5 m long and 1200 mm^2 cross-sectional area hangs vertically and has collar securely fixed at its lower end. Find the stress induced in the bar when a weight of 500N Falls from a Height of 100 mm on a collar by taking E= 150 G Pa. Also find the strain energy Stored in bar.**

Ans.:

Given Data:

$l = 1.5\ m = 1500\ mm$

$A = 1200\ mm^2$

$W = 500\ N$

$h = 100\ mm$

$E = 150\ GPa = 150 \times 10^3\ {N}/{mm^2}$

$U, \sigma_{inst} = ?$

Instantaneous stress induced in elastic rod due to Impact load

$$we\ have\ \ \sigma_{Inst} = \left(\frac{W}{A}\right) + \sqrt{\left(\frac{W}{A}\right)^2 + \left(\frac{2EWh}{Al}\right)} = \left(\frac{500}{1200}\right) + \sqrt{\left(\frac{500}{1200}\right)^2 + \left(\frac{2 \times 500 \times 100 \times 150 \times 10^3}{1200 \times 1500}\right)}$$

$$\sigma_{Inst} = \left(\frac{500}{1200}\right) + \sqrt{\left(\frac{500}{1200}\right)^2 + \left(\frac{2 \times 500 \times 100 \times 150 \times 10^3}{1200 \times 1500}\right)}$$

$$= \left(\frac{500}{1200}\right) + \sqrt{8333.506944} = 91.70\ {N}/{mm^2}$$

$\therefore$ Instantaneous stress induced in elastic rod due to Impact load $\boldsymbol{\sigma_{Inst} = 91.70\ {N}/{mm^2}}$

Strain Energy Produced due to Impact load:

Now we have strain *energy*

$$U = \frac{\sigma^2}{2E} Al = \frac{91.70^2 \times 1200 \times 1500}{2 \times 150 \times 10^3} = 50453.34\ Nmm\ say\ 50.453\ J$$

$$\therefore\ \boldsymbol{U = 50.453\ J}$$

Q. A mild steel bar of 3m long and 50 mm diameter hangs vertically and has a collar securely attached at the lower end. Find the maximum stress induced, when a weight of 25 KN falls on the collar from a height of 20 mm. Take young's modulus of elasticity for mild steel as 220 GN/m^2. [2017, 2016]

Ans.:

Given Data:

$l = 3000\ mm$

$d = 50\ mm,$

$\sigma_{Maximum} = ?$

$W = 25\ KN = 25000\ N$

$h = 20mm$

$E = 220G\ {N}/{m^2} = 220 \times 10^3\ {N}/{mm^2}$

$$Area\ of\ Steel\ bar,\quad A_S = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 50^2 = 1963.49\ mm^2$$

Instantaneous stress induced in elastic rod due to Impact load

$$we\ have\ \sigma_{Inst} = \left(\frac{W}{A}\right) + \sqrt{\left(\frac{W}{A}\right)^2 + \left(\frac{2EWh}{Al}\right)} = \left(\frac{25000}{3000}\right) + \sqrt{\left(\frac{25000}{1963.49}\right)^2 + \left(\frac{2 \times 25000 \times 20 \times 220 \times 10^3}{1963.49 \times 3000}\right)}$$

$$\sigma_{Inst} = \left(\frac{25000}{1963.49}\right) + \sqrt{\left(\frac{25000}{1963.49}\right)^2 + \left(\frac{2 \times 25000 \times 100 \times 220 \times 10^3}{1963.49 \times 3000}\right)}$$

$$= \left(\frac{25000}{1963.49}\right) + \sqrt{37417.90} = 206.16\ {}^{N}/_{mm^2}$$

∴ Instantaneous stress induced in elastic rod due to Impact load $\boldsymbol{\sigma_{Inst} = 206.16\ {}^{N}/_{mm^2}}$

Important Formulae

i. $stress\ \sigma = \frac{P}{A}\quad P - applied\ load, A - area\ of\ crosssection$

ii. $Strain\ e = \frac{\delta l}{l}, l - Original\ Length, \delta l - change\ in\ length$

iii. $E = \frac{\sigma}{e},\ E -\ young's Modulus\ of\ Elasticity$

iv. $E = 2G(1 + \mu),\ G - Modulus\ of\ rigidity,\ \mu - Poisson's\ ratio$

v. $E = 3K(1 - 2\mu),\ K - Bulk\ modulus\ of\ elasticity$

vi. Principle of superposition

$$\delta l = \frac{1}{AE}(P_1 l_1 + P_2 l_2 + P_3 l_3)$$

$$P_1, P_2, P_3 - Loads\ acting\ on\ induvidual\ sections\ of\ bar\ as\ 1, 2, 3\ Respectively$$

$$l_1, l_2, l_3 - length\ of\ induvidual\ sections\ of\ bar\ as\ 1, 2, 3\ respectivel$$

vii. Composite Bar : $e_1 = e_2$ .... $equal\ deformation$

$$applied\ load,\ P = P_1 + P_2 = \sigma_1 A_1 + \sigma_2 A_2$$

viii. $Temp\ stress\quad \sigma = \alpha.\Delta T.E$

ix. $Temp\ stress\ in\ compound\ bar$

$$P_1(Tensile\ Load) = P_2(Compressive\ Load)$$

$$\frac{\sigma_1}{E_1} + \frac{\sigma_2}{E_2} = (\alpha_1 - \alpha_2)\Delta T, \qquad if\ \alpha_1 > \alpha_2$$

x. $Strain\ energy,\ U = \frac{\sigma^2}{2E} \times Al$

xi. $Stress\ due\ to\ gradual\ load\ \sigma = \frac{P}{A}$

xii. $Stress\ due\ to\ suddenly\ applied\ load\ \ \sigma = 2\frac{P}{A}$

xiii. $Stress\ due\ to\ impact\ loading,\ \sigma_{Inst} = (\frac{W}{A}) + \sqrt{(\frac{W}{A})^2 + (\frac{2EWh}{Al})}$

Chapter-2
Thin cylindrical and spherical shell

What is a thin cylinder? (How will you consider a vessel thin? Give two example.) [2017 S]

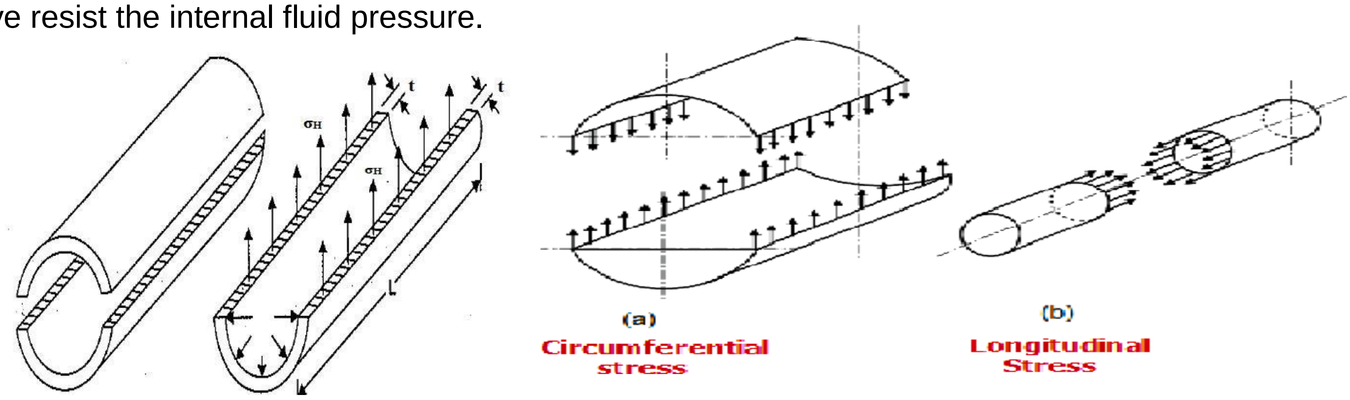
❖ Thin cylinder:

Definition:

A cylinder is said to be a thin cylinder when it's thickness of wall is less than 1/10th to 1/20th of its diameter.

❖ **Failure in thin cylindrical Shell due to internal fluid pressure:**

In our practical life we come across different types of thin cylinder are used to store oil, water or steam and also for the supply from one place to another place. During store/supplyof fluid
The pressure of fluid exerts force on the internal surface of cylinder. The material of cylinder should have resist the internal fluid pressure.



It is observed that if the pressure of fluid is more than the capacity of material of cylinder. It causes the bursting of cylinder as

i. The cylinder may split up into two troughs
ii. It may split up in to two cylinder

There are two types of forces are acting along its diameter and length which induces the stresses in shell

i. Circumferential force
ii. Longitudinal force

Q. Derive the expression for hoop and longitudinal stresses for a thin cylindrical shell under internal pressure. [2011, 2012, 2013, 2014w, 2015, 2016 W]

Let's consider a thin cylinder is used to supply the fluid under pressure

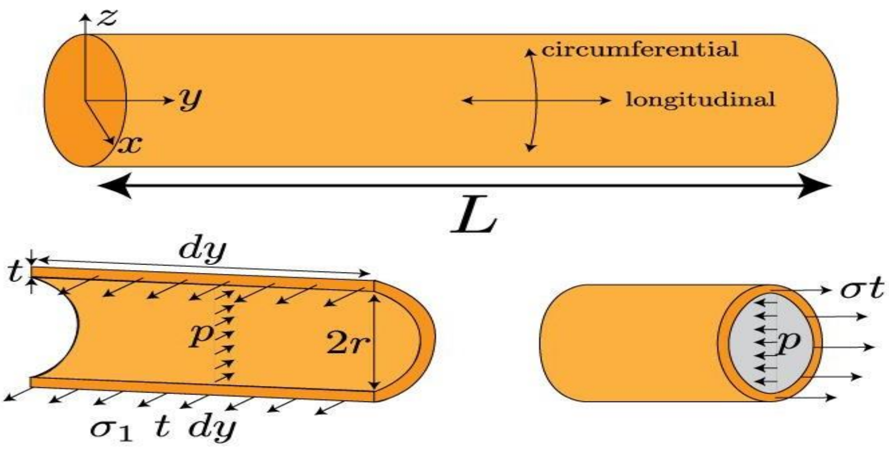
P – pressure of fluid inside the cylinder
t – thickness of cylinder
l – length of cylinder
d – diameter of cylinder
σ_C – Hoop stress or circumferential stress induced in cylinder
σ_l – Longitudinal stress induced in cylinder

Circumferential stress or Hoop Stress:

It is defined as the stress which induced in a direction to the circumferential of thin cylinder

It causes bursting of cylinder in to two troughs as shown by fig.

For the safety condition of cylinder bursting force offered by Internal pressure should be equal to resistance force offered by the material thin cylinder



Mathematically

$$\text{Bursting Force} = \text{Resisting force offered by cylinder}$$

$$\Rightarrow P.d.l = 2.t.l.\sigma_C$$

$$\boldsymbol{\sigma_c = \frac{Pd}{2t}} \ldots \ldots\ldots\ldots\ldots \text{ Hoop Stress induced in cylinder}$$

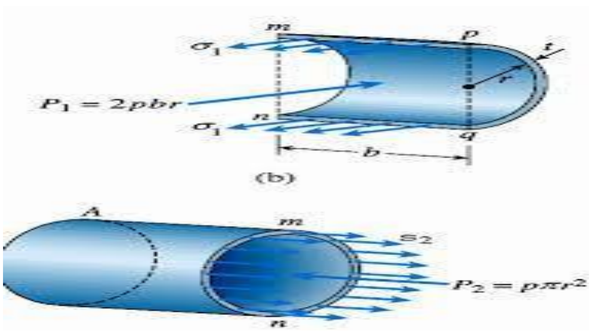
Unit: it is expressed in $N/_{mm^2}$, Pa, MPa, G Pa

Longitudinal Stress:

It is defined as the stress which is developed along the longitudinal direction of cylinder under the action of internal fluid Pressure. It causes the cylinder may split up into two numbers as shown in figure.

For the safety condition of cylinder bursting force offered by Internal pressure should be equal to resistance force offered by the material thin cylinder

Mathematically



$$\text{Bursting(Pressure) Force} = \text{Resisting force offered by cylinder}$$

$$\Rightarrow P \times \frac{\pi}{4} \times d^2 = \sigma_l \times \pi d.t$$

$$\boldsymbol{\sigma_l = \frac{Pd}{4t}} \ldots \ldots\ldots\ldots\ldots \text{ Longitudinal Stress induced in cylinder}$$

Unit: it is expressed in $N/_{mm^2}$, Pa, MPa, G Pa

1. **State the expression of hoop and longitudinal stress.**

[2009(w), 2010, 2011, 2012, 2013, 2014, 2016(O&N) 2017 W (N)]

Ans. Hoop Stress: It is defined as the stress which induced in a direction to the circumferential of thin cylinder it is denoted by symbol $'\sigma_c'$

$$\sigma_c = \frac{Pd}{2t}$$

Longitudinal Stress: It is defined as the stress which is developed along the longitudinal direction of cylinder under the action of internal fluid Pressure.it is denoted by $'\sigma_l'$

$\mathrm{P} - \text{pressure of fluid inside the cylinder}$
$\mathrm{t} - \text{thickness of cylinder}$
$\mathrm{d} - \text{diameter of Cylinder}$

❖ **Strain Induced in thin cylindrical shell:**

As above we discussed that due to high internal fluid pressure more than the capacity of cylindrical shell

There are deformations carried out in cylinder along it's diameter and length during bursting of cylinder.

There are following types strains are produced

i. Hoop/ Circumferential Strain
ii. Longitudinal Strain
iii. Volumetric strain

Let's consider a thin cylinder is subjected to internal fluid pressure which causes the deformation of cylinder

$\sigma_C - \text{Hoop stress induced in cyinder}$
$\sigma_l - \text{longitudinal tress induced in cyinder}$
$\mathrm{E} - \text{Modulus of elasticity for material of thin cylinder}$
$\delta\mathrm{d} - \text{change in diameter of cylinder}$
$\delta\mathrm{l} - \text{change in length of cylinder}$
$\mathrm{d} - \text{Original diameter of cylinder}$
$\mathrm{l} - \text{Original length of cylinder}$
$\mu - \text{Poisson's ratio}$
$\mathrm{V} - \text{Original Volume of cylinder}$
$\delta\mathrm{V} - \text{change in volume of cylinder}$

i. Hoop Strain:

It is defined as the ratio of change in diameter to original diameter of cylinder under action of internal fluid pressure.

It is denoted by symbol $'e_c'$

Mathematically,

$$\text{Hoop strain,} \quad e_C = \frac{\text{Change in Diameter}}{\text{Original Diameter}} = \frac{\delta d}{d}$$

$$e_C = \frac{\delta d}{d}$$

Now $\text{Hoop strain} = \text{longitudinal strain along Diameter} - \text{lateral strain along length}$

$$\text{hoop Strain} \quad e_C = \frac{\sigma_C}{E} - \mu\frac{\sigma_l}{E} = \frac{\sigma_C}{E} - \mu\frac{\sigma_c}{2E} = \frac{\sigma_C}{E}(1 - \frac{1}{2} \times \mu)$$

$$e_C = \frac{\sigma_C}{E}(1 - \frac{1}{2} \times \mu) \text{......................Hoop Strain}$$

Unit: it is unit less

ii. Longitudinal Strain:

It is defined as the ratio of change in length to original length of cylinder under action of internal fluid pressure.

It is denoted by symbol $'e_l'$

Mathematically,

$$\text{Longitudinal strain,} \quad e_l = \frac{\text{Change in Length}}{\text{Original Length}} = \frac{\delta l}{l}$$

$$e_l = \frac{\delta l}{l}$$

Now $\text{Longitudinal strain} = \text{longitudinal strain along length} - \text{lateral strain along Diameter}$

$$\text{Longitudinal Strain} \quad e_l = \frac{\sigma_l}{E} - \mu\frac{\sigma_c}{E} = \frac{\sigma_C}{2E} - \mu\frac{\sigma_c}{E} = \frac{\sigma_c}{E}(\frac{1}{2} - \mu)$$

$$e_l = \frac{\sigma_c}{E}(\frac{1}{2} - \mu) \ldots\ldots\ldots\ldots\ldots.. \text{Longitudinal Strain}$$

Unit: It is unit less

2. **Define hoop stress & strain.** **[2012, 2014wbp]**

Ans.

Hoop Stress:

It is defined as the stress which induced in a direction to the circumferential of thin cylinder it is denoted by symbol $'\sigma_c'$

$$\sigma_c = \frac{Pd}{2t}$$

Where

$P -$ pressure of fluid inside the cylinder

$t -$ thickness of cylinder

$d -$ diameter of Cylinder

Hoop Strain:

It is defined as the ratio of change in diameter to original diameter of cylinder under action of internal fluid pressure.

It is denoted by symbol $'e_c'$

Mathematically,

$$e_C = \frac{\text{Change in Diameter}}{\text{Original Diameter}} = \frac{\delta d}{d}$$

$$e_C = \frac{\delta d}{d}$$

❖ **Change in Dimension of Cylinder:**

Under action of internal fluid pressure the cylinder deforms along Dimeter wise as well as longitudinally

i. Change in Diameter ($'\delta d'$):

We know that Hoop Strain

$$e_C = \frac{\sigma_C}{E}(1 - \frac{1}{2} \times \mu) \text{................ (i)}$$

$$e_C = \frac{\delta d}{d} \text{... .... (ii)}$$

Now equating the above equation (i) and (ii) we have

$$e_C = \frac{\delta d}{d} = \frac{\sigma_C}{E}(1 - \frac{1}{2} \times \mu)$$

$$\Rightarrow \frac{\delta d}{d} = \frac{\sigma_C}{E}(1 - \frac{1}{2} \times \mu)$$

$$\Rightarrow \delta d = d \times \frac{\sigma_C}{E}(1 - \frac{1}{2} \times \mu) = \frac{Pd^2}{2t.E}(1 - \frac{1}{2} \times \mu)$$

$$\Rightarrow \boldsymbol{\delta d = \frac{Pd^2}{2t.E}(1 - \frac{1}{2} \times \mu)} \text{ ....Change in diameter of Cylinder}$$

ii. Change in Length (δl):

We know that longitudinal Strain

$$e_l = \frac{\delta l}{l} \dots\dots\dots\dots\dots\dots\dots (i)$$

$$e_l = \frac{\sigma_c}{E}(\frac{1}{2} - \mu) \text{.................. (ii)}$$

Now equating both equations

$$e_l = \frac{\delta l}{l} = \frac{\sigma_c}{E}(\frac{1}{2} - \mu)$$

$$\Rightarrow \frac{\delta l}{l} = \frac{\sigma_c}{E}(\frac{1}{2} - \mu)$$

$$\Rightarrow \delta l = l.\frac{\sigma_c}{E}(\frac{1}{2} - \mu) = \frac{Pdl}{2tE}(\frac{1}{2} - \mu)$$

$$\Rightarrow \boldsymbol{\delta l = \frac{Pdl}{2tE}(\frac{1}{2} - \mu)} \dots\dots\dots\dots\dots\dots \textbf{. Change in length of Cylinder}$$

3. **A boiler shell 3 m long, 50 cm in diameter and 1.25 cm thick is at atmospheric pressure. Calculate the circumferential and longitudinal stress in the boiler plate if the shell is subjected to an internal pressure of 2MN/m^2.also calculate the change in dimensions of the shell Take E = 200 GN/m^2 and Poisson's ratio as 0.25. [2018 S (N)]**

Ans.:

Given Data

$l = 3m = 3000 \text{ mm}$

$d = 50cm = 500mm$

$t = 1.25 \text{ cm} = 12.5 \text{ mm}$

$P = 2M\frac{N}{m^2} = 2\frac{N}{mm^2}$

$\mu = 0.25$

$E = 200G\frac{N}{m^2} = 200 \times 10^3 \frac{N}{m^2}$

$\sigma_c, \sigma_l = ?$

$\delta d, \delta l = ?$

Circumferential Stress:

$$\sigma_c = \frac{Pd}{2t} = \frac{2 \times 500}{2 \times 12.5} = 40\frac{N}{mm^2} \boldsymbol{\sigma_c = 40\frac{N}{mm^2}}$$

Longitudinal Stress:

$$\sigma_l = \frac{Pd}{4t} = \frac{2 \times 500}{4 \times 12.5} = 20\frac{N}{mm^2}, \qquad \boldsymbol{\sigma_l = 20\frac{N}{mm^2}}$$

Change in diameter of Cylinder $'\delta d'$:

we know that

$$\delta d = \frac{Pd^2}{2t.E}(1 - \frac{1}{2} \times \mu)$$

$$\delta d = \frac{2 \times 500^2}{2 \times 12.5 \times 200 \times 10^3}(1 - \frac{1}{2} \times 0.25)$$

$$\mathbf{\delta d = 0.0875\ mm}$$

Change in length of Cylinder'δl':

We know that

$$\delta l = \frac{Pdl}{2tE}(\frac{1}{2} - \mu)$$

$$\delta l = \frac{2 \times 500 \times 3000}{2 \times 12.5 \times 200 \times 10^3}(\frac{1}{2} - 0.25) = 0.15\ mm$$

$$\mathbf{\delta l = 0.15\ mm}$$

4. **A thin cylindrical shell is 3m in length, inside diameter is 1m, Thickness of metal 10 mm, internal pressure of fluid is 1.5 MPa. Calculate Hoop Stress, longitudinal stress, change in diameter and length. Take poison's ratio as 0.25,$\mathbf{E = 200G\frac{N}{m^2}}$[2017 W (N**

   Ans.:

Given Data

$l = 3m = 3000\ mm$

$d = 1m = 1000mm$

$t = 10\ mm$

$P = 1.5\ M\frac{N}{m^2} = 1.5\frac{N}{mm^2}$

$\mu = 0.25$

$E = 200G\frac{N}{m^2} = 200 \times 10^3\ \frac{N}{m^2}$

$\sigma_c, \sigma_l = ?$

$\delta d, \delta l = ?$

Circumferential Stress:

$$\sigma_c = \frac{Pd}{2t} = \frac{1.5 \times 1000}{2 \times 10} = 75\ \frac{N}{mm^2}\ \boldsymbol{\sigma_c = 75\ \frac{N}{mm^2}}$$

Longitudinal Stress:

$$\sigma_l = \frac{Pd}{4t} = \frac{1.5 \times 1000}{4 \times 10} = 37.5\ \frac{N}{mm^2}, \qquad \boldsymbol{\sigma_l = 37.5\frac{N}{mm^2}}$$

Change in diameter of Cylinder'δd':

We know that

$$\delta d = \frac{Pd^2}{2t.E}(1 - \frac{1}{2} \times \mu)$$

$$\delta d = \frac{1.5 \times 1000^2}{2 \times 10 \times 200 \times 10^3}(1 - \frac{1}{2} \times 0.25) = 0.3281mm$$

$$\mathbf{\delta d = 0.3281\ mm}$$

Change in length of Cylinder'δd':

We know that

$$\delta l = \frac{Pdl}{2tE}(\frac{1}{2} - \mu)$$

$$\delta l = \frac{1.5 \times 1000 \times 3000}{2 \times 10 \times 200 \times 10^3}(\frac{1}{2} - 0.25) = 0.2812mm$$

$$\mathbf{\delta l = 0.2812mm}$$

❖ **Volumetric Strain:**

It is defined as the ratio of change in Volume to original Volume of thin cylinder under action internal fluid pressure. It is denoted by symbol $'e_V'$

Mathematically

Volumetric Strain

$$e_V = \frac{\text{Change in Volume}}{\text{original Volume}} = \frac{\delta V}{V}$$

$$e_V = \frac{\delta V}{V}$$

We have Original volume of Cylinder

$$V = \frac{\pi}{4} d^2 l$$

Now change in volume of cylinder will be obtained by Partial derivation of Volume with respect to length and diameter of cylinder.

$$\frac{\delta(V)}{\delta l.\delta d} = \frac{\delta\left(\frac{\pi}{4}d^2 l\right)}{\delta l.\delta d} = \frac{\pi}{4}[2d.l + 1.d^2]$$

$$\boldsymbol{\delta V = \frac{\pi}{4}[2d.l\delta d + 1.d^2\delta l.]}$$

Now Volumetric Strain

$$e_V = \frac{\delta V}{V} = \frac{\frac{\pi}{4}[2d.l\delta d + 1.d^2\delta l.]}{\frac{\pi}{4}d^2 l} = 2\frac{\delta d}{d} + \frac{\delta l}{l}$$

$$e_V = 2\frac{\delta d}{d} + \frac{\delta l}{l} = 2e_C + e_l$$

$$\boldsymbol{e_V = 2e_C + e_l} \text{.......................................Volumetric Strain in cylinder}$$

❖ **Change in Volume of Cylinder:**

We have volumetric Strain

$$\boldsymbol{e_V = 2e_C + e_l}$$

We have hop strain and longitudinal strain induced in thin cylinder and now putting the value from below equation (I) and (ii)

$$e_l = \frac{\sigma_c}{E}\left(\frac{1}{2} - \mu\right)\text{.................(i)}$$

$$e_C = \frac{\sigma_C}{E}\left(1 - \frac{1}{2}\times\mu\right)\text{.................. (ii)}$$

Now

$$\boldsymbol{e_V = 2e_C + e_l = 2}\frac{\sigma_C}{E}\left(1 - \frac{1}{2}\times\mu\right) + \frac{\sigma_c}{E}\left(\frac{1}{2} - \mu\right)$$

$$\Rightarrow \boldsymbol{e_V} = \left[\frac{\sigma_C}{E}\left(2\left(1 - \frac{1}{2}\times\mu\right) + \left(\frac{1}{2} - \mu\right)\right)\right]$$

$$\Rightarrow \boldsymbol{e_V} = \frac{\sigma_C}{E}\left(2 - \mu + \frac{1}{2} - \mu\right) = \frac{\sigma_C}{E}\left(2 + \frac{1}{2} - 2\mu\right)$$

$$\Rightarrow \boldsymbol{e_V} = \frac{\sigma_C}{E}\left(2 + \frac{1}{2} - 2\mu\right) = \frac{\sigma_C}{E}\left(\frac{5 - 4\mu}{2}\right)$$

$$\Rightarrow \boldsymbol{e_V} = \frac{\sigma_C}{E}\left(\frac{5 - 4\mu}{2}\right) = \frac{Pd}{4tE}(5 - 4\mu)$$

$$\Rightarrow \boldsymbol{e_V} = \frac{Pd}{4tE}(5 - 4\mu)\text{........................(iii)}$$

We have

$$\boldsymbol{e_V} = \frac{\delta V}{V}\text{ ... . (iv)}$$

Now equating equation (iii) and (IV)

$$\Rightarrow \mathbf{e_V} = \frac{\delta V}{V} = \frac{Pd}{4tE}(5-4\mu)$$

$$\Rightarrow \delta V = V.\frac{Pd}{4tE}(5-4\mu) = \frac{\pi}{4}d^2l.\frac{Pd}{4tE}(5-4\mu)$$

$$\Rightarrow \boldsymbol{\delta V = \frac{\pi P d^3 l}{16tE}(5-4\mu)} \ldots\ldots\ldots\ldots \textbf{Change in Volume of thin Cylinder}$$

5. A cylindrical vessel 2 m long and 500 mm in dia. With 10 mm thick plate is subjected to an internal pressure of 3 M Pa. calculate the change in volume of vessel. Take E is 200 GPa, Poisson's ratio 0.3 for vessel material. [2016w, 2013w, 2009w]

Ans.:

Given Data

$l = 2\text{m} = 2000\ \text{mm}$

$d = 500 = 500\text{mm}$

$t = 10\ \text{mm}$

$P = 3\ M\frac{N}{m^2} = 3\frac{N}{mm^2}$

$\mu = 0.3$

$E = 200G\frac{N}{m^2} = 200\times10^3\frac{N}{m^2}$

$\delta V = ?$

i. Circumferential Stress:

$$\sigma_c = \frac{Pd}{2t} = \frac{3\times500}{2\times10} = 75\frac{N}{mm^2}\ \boldsymbol{\sigma_c = 75\frac{N}{mm^2}}$$

Hoop strain:

We have

$$e_C = \frac{\sigma_C}{E}(1-\frac{1}{2}\times\mu) = \frac{75}{200\times10^3}(1-\frac{1}{2}\times0.3) = 0.00031875$$

$$\mathbf{e_C = 0.00031875}$$

ii. Longitudinal Strain:

We have

$$e_l = \frac{\sigma_c}{E}(\frac{1}{2}-\mu) = \frac{75}{200\times10^3}(\frac{1}{2}-0.3) = 0.000075$$

$$\mathbf{e_l = 0.000075}$$

Now Volume of body

$$V = \frac{\pi}{4}d^2l = \frac{\pi}{4}\times500^2\times2000 = 3926990817\ mm^3$$

$$\mathbf{V = 3926990817\ mm^3}$$

iii. Change in Volume:

We volumetric strain

$$e_V = 2e_C + e_l = 2\times0.00031875 + 0.000075 = 0.000712500$$

$$\mathbf{e_V = 0.000712500}$$

We have

$$e_V = \frac{\delta V}{V}$$

$$\Rightarrow \delta V = e_V\times V = 0.000712500\times3926990817 = 2797980.757mm^3$$

$$\boldsymbol{\delta V = 2797980.757mm^3} \ldots\ldots\ldots\ldots\ldots\ldots.. \textbf{Change in Volume of Cylinder}$$

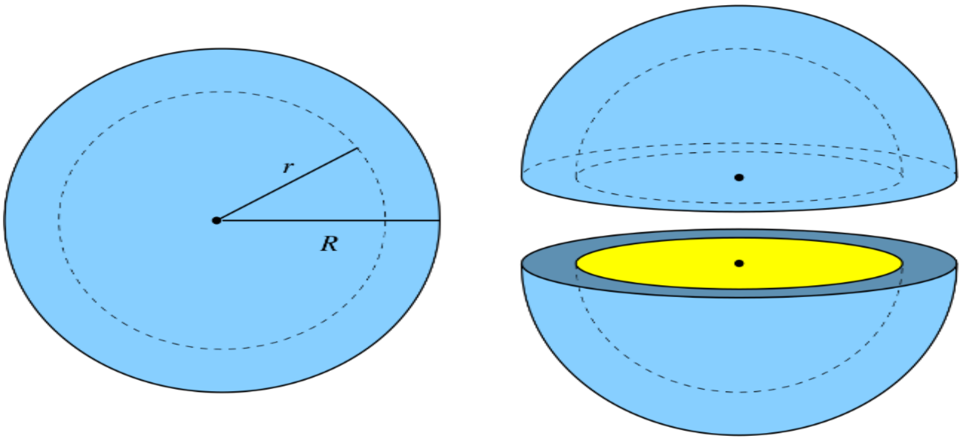
❖ Thin Spherical Shell:

A sphere is the theoretical ideal shape for a vessel that resists internal pressure.

i. Pressure vessels are closed structures containing liquids or gases under pressure .Example include tanks, pipes, pressurized cabins etc.

ii. When pressure vessels have walls that are thin in comparison to their radii and length. In the case of thin walled pressure vessels of spherical shape the ratio of radius ‘r’ to wall thickness’t’ is greater than 10.

iii. To determine the stresses in a spherical vessel let us cut through the sphere on a vertical diameter plane and isolate half of the shell and its fluid contents as a single free body. Acting on this free body are the tensile stress in the wall of the vessel and the fluid pressure.



Stresses induced in Thin Spherical Shell:

A thin Spherical shell is used to store oil or water under pressure.

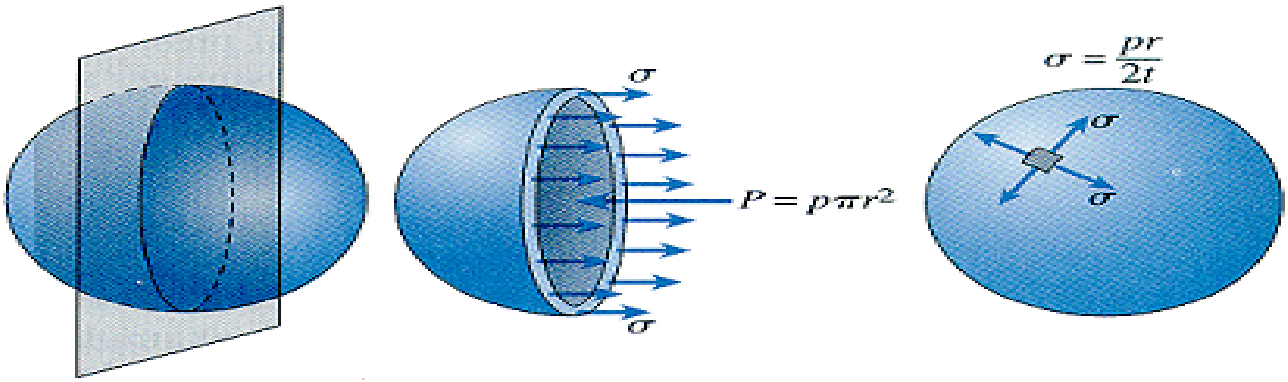
Let $P -$ Pressure of fluid inside the shell

$t -$ thickness of shell

$d -$ diameter of spherical

Circumferential Stress:

Due to increase of pressure of fluid inside shell, the spherical shell is converted into two hemisphere as shown as in figure



For safety condition of sphere.

Total Bursting Force $=$ Resisting force given by shell

$\Rightarrow P \times \frac{\pi}{4} \times d^2 = \pi d.t.\sigma_c$

$$\sigma_C = \frac{\mathbf{Pd}}{\mathbf{4t}} \ldots\ldots\ldots\ldots\ldots\ldots \textbf{Circumferential Stress}$$

❖ **A spherical shell of 800 mm dia is subjected to an internal fluid pressure of 2 MPa. What should be the minimum thickness of shell if the tensile stress is not to exceed 50 M Pa [2017 w]**

Ans.

Given Data:

$d = 800 \text{ mm}$

$P = 2\text{MPa} = 2 \text{ N/mm}^2$

$\sigma_C = 50 \text{ MPa} = 50 \text{ N/mm}^2$

$t = ?$

We have Hoop Stress

$$\sigma_C = \frac{Pd}{4t} = \frac{2 \times 800}{4 \times t} = 50$$

$$\Rightarrow t = \frac{2 \times 800}{4 \times 50} = 8 \text{ mm}$$

$$\Rightarrow \mathbf{t = 8\ mm} \text{........................ thickness of Spherical Shell}$$

❖ Change in Diameter and Volume of a thin Spherical Shell due to Internal Pressure:

Let's consider a thin spherical shell is subjected to internal pressure.

P – internal pressure inside sphere

d – diameter of sphere

t – thickness of sphere

σ_C – hoop Stress induced

E – young's Modulus for material of Sphere

μ – poisson's ratio

V – Original Volume of sphere

δd – Change in diameter

δV – Change of Volume

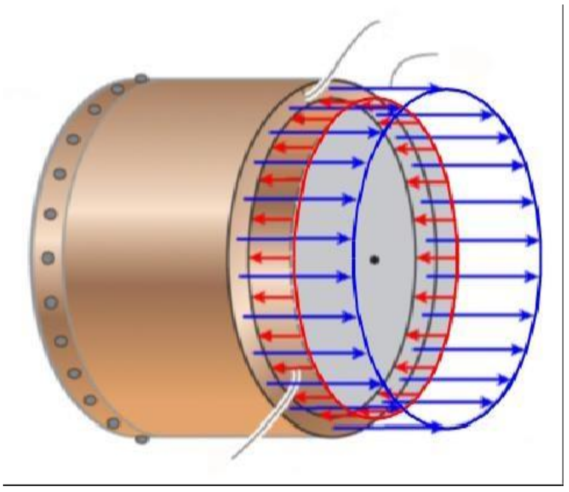
Hoop Strain:

It is the ratio of change in diameter to original Diameter of thin cylinder under internal fluid pressure.

It is denoted by symbol 'e_C'

$$e_C = \frac{\text{change in diameter}}{\text{Original diameter}} = \frac{\delta d}{d}$$

$$\mathbf{e_C = \frac{\delta d}{d}} \text{ .... } \mathbf{(i)}$$



Hoop strain is given by

e_C = Longitudinal Strain along X – direction

+ Lateral strain along transverse y

– direction

$$e_C = e_x + \mu . e_y = \frac{\sigma_C}{E} - \mu \frac{\sigma_C}{E}$$

$$\Rightarrow \mathbf{e_C} = \frac{\boldsymbol{\sigma_C}}{\mathbf{E}}(\mathbf{1}-\boldsymbol{\mu}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{Hoop Strain iduced in bar}$$

Change in Diameter $'\delta d'$:

From equation (i)

$$e_C = \frac{\delta d}{d}$$

$$\Rightarrow \boldsymbol{\delta d} = \mathbf{e_C.\,d}$$

$$\Rightarrow \boldsymbol{\delta d} == \mathbf{d.}\frac{\boldsymbol{\sigma_C}}{\mathbf{E}}(\mathbf{1}-\boldsymbol{\mu}) \ldots\ldots\ldots\ldots\ldots\ldots \text{Change in diameter of sphere}$$

Q. **Find out the change in volume of a thin spherical shell due to internal pressure.**

Volumetric Strain:

It is defined as the ratio of change in Volume to original Volume of thin cylinder under action internal fluid pressure. It is denoted by symbol $'e_V'$

Mathematically Volumetric Strain

$$e_V = \frac{\text{Change in Volume}}{\text{original Volume}} = \frac{\delta V}{V}$$

$$e_V = \frac{\delta V}{V} \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Original Volume of Sphere

$$V = \frac{\pi}{6}d^3$$

Change in Volume of Sphere will be obtained by partial derivative of volume with respect of diameter

$$\frac{\delta V}{\delta d} = \frac{\delta\left(\frac{\pi}{6}d^3\right)}{\delta d}$$

$$\delta V = \frac{\pi}{6}(3d^2)\delta d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{Change in Volume of Sphere}$$

Now Volumetric Strain

$$e_V = \frac{\delta V}{V} = \frac{\frac{\pi}{6}(3d^2)\delta d}{\frac{\pi}{6}d^3} = 3\frac{\delta d}{d} = 3e_C$$

$$e_V = 3e_C = 3\frac{\sigma_C}{E}(1-\mu) \ldots\ldots\ldots\ldots\ldots\ldots (iii)\ .\text{Volumetric strain}$$

Now change in Volume of Sphere

$$e_V = 3e_C = 3 \times \frac{\sigma_C}{E}(1-\mu)$$

$$e_V = 3 \times \frac{\sigma_C}{E}(1-\mu)$$

$$\Rightarrow \frac{\delta V}{V} = 3 \times \frac{\sigma_C}{E}(1-\mu)$$

$$\Rightarrow \boldsymbol{\delta V} = \mathbf{3} \times \frac{\boldsymbol{\sigma_C}}{\mathbf{E}}(\mathbf{1}-\boldsymbol{\mu}) \times \mathbf{V} \ldots\ldots\ldots\ldots\ldots\ldots \text{Change in volume of Sphere}$$

Q. Calculate the increase in volume of a spherical shell 1m in diameter and 1 cm thick when it is subjected to an internal pressure of 1.6 MN/m^2. Take E = 200 GN/m^2 and poison's ratio as 0.3.

Ans.:

Given Data:

$d = 1m = 1000$ mm
$t = 1cm = 10mm$
$P = 1.6\ M\ N/m^2 = 1.6\ N/mm^2$
$E = 200\ G\ N/m^2 = 200 \times 10^3\ N/mm^2$
$\mu = 0.3$
We have Hoop Stress

$$\sigma_C = \frac{Pd}{4t} = \frac{1.6 \times 1000}{4 \times 10} = 40\ N/mm^2$$

Original Volume of Sphere

$$V = \frac{\pi}{6} d^3 = \frac{\pi}{6} \times 1000^3 = 523598775.5\ mm^3$$

Hoop strain induced in Sphere

$$e_C = \frac{\sigma_C}{E}(1 - \mu)$$

$$= \frac{40}{200 \times 10^3}(1 - 0.3) = 0.00014$$

$$e_C = 0.00014$$

We know that volumetric strain

$$e_V = 3e_C = 3 \times 0.00014 = 0.00042$$

Change in Volume of Sphere
We know that Volumetric Strain

$$e_V = \frac{\delta V}{V} = 0.00042$$

$$\Rightarrow \delta V = 0.00042 \times V = 0.00042 \times 523598775.5 = 219911.48\ mm^3$$

$$\Rightarrow \boldsymbol{\delta V = 219911.48\ mm^3} \text{.................................. increase in Volume of Sphere}$$

Q. **A spherical shell 300 mm diameter with a wall thickness 8mm is filled with a fluid at atmospheric pressure. If an additional 6000 mm^3 of fluid is pumped into cylinder, Find the pressure exerted by fluid on the wall of sphere. Find also the hoop stress induced Take E = 200 GN/m^2 and poison's ratio as 0.3.**

Ans.:

Given Data:

$d = 300$ mm
$t = 08$ mm
$E = 200\ G\ N/m^2 = 200 \times 10^3\ N/mm^2$
$\mu = 0.3$
$\delta V = 6000\ mm^3$
$\sigma_C, P = ?$
Original Volume of Sphere

$$V = \frac{\pi}{6} d^3 = \frac{\pi}{6} \times 300^3 = 14137166.94\ mm^3$$

We know that Volumetric Strain

$$e_V = \frac{\delta V}{V} = \frac{6000}{14137166.94} = 0.000424413$$

We know that volumetric strain

$$e_V = 3e_C$$

$$\Rightarrow e_C = \frac{e_V}{3} = \frac{0.000424413}{3} = 0.000141471$$

$$\Rightarrow \boldsymbol{e_C = 0.000141471} \text{.......................... Hoop Strain in Sphere}$$

Hoop strain induced in Sphere

$$e_C = \frac{\sigma_C}{E}(1-\mu) = 0.000141471$$

$$\Rightarrow \sigma_C = \frac{0.000141471}{(1-0.3)} \times 200 \times 10^3$$

$$\boldsymbol{\sigma_C = 40.42\ \mathrm{N/mm^2}} \ldots\ldots.. \textbf{ Hoop Stress induced in Sphere}$$

We have Hoop Stress

$$\sigma_C = \frac{Pd}{4t} = 40.42\ \mathrm{N/mm^2}$$

$$\Rightarrow P = \frac{40.42 \times 4 \times 8}{300} = 4.31\ \mathrm{N/mm^2}$$

$$\mathbf{P = 4.31\ N/mm^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{internal fluid pressure in Sphere}$$

Important Formulae

Thin Cylinder:

$$\boldsymbol{\sigma_l = \frac{Pd}{4t}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{Longitudinal Stress induced } \textit{in cylinder}$$

$$\boldsymbol{\sigma_c = \frac{Pd}{2t}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textit{Hoop Stress induced in cylinder}$$

$$e_C = \frac{\delta d}{d}$$

$$e_C = \frac{\sigma_C}{E}\left(1 - \frac{1}{2} \times \mu\right) \ldots\ldots\ldots\ldots\ldots.. \textit{Hoop Strain}$$

$$e_l = \frac{\delta l}{l}$$

$$e_l = \frac{\sigma_c}{E}\left(\frac{1}{2} - \mu\right) \ldots\ldots\ldots\ldots\ldots.. \textit{Longitudinal Strain}$$

$$\boldsymbol{\delta d = \frac{Pd^2}{2t.E}\left(1 - \frac{1}{2} \times \mu\right)} \ldots\ldots\ldots\ldots \textit{Change in diameter of Cylinder}$$

$$\boldsymbol{\delta l = \frac{Pdl}{2tE}\left(\frac{1}{2} - \mu\right)} \ldots\ldots\ldots\ldots \text{Change in length of Cylinder}$$

$$e_V = \frac{\delta V}{V}$$

$$\boldsymbol{e_V = 2e_C + e_l} \ldots\ldots\ldots\ldots \text{Volumetric Strain in cylinder}$$

$$\boldsymbol{\delta V = \frac{\pi P d^3 l}{16tE}(5 - 4\mu)} \ldots\ldots\ldots\ldots \text{Change in Volume of thin Cylinder}$$

❖ Thin Spherical Shell:

$$\sigma_C = \frac{Pd}{4t} \ldots\ldots\ldots\ldots \text{Circumferential Stress}$$

$$e_C = \frac{\sigma_C}{E}(1 - \mu) \ldots\ldots\ldots\ldots \text{Hoop Strain iduced in bar}$$

$$\boldsymbol{\delta d == d.\frac{\sigma_C}{E}(1 - \mu)} \ldots\ldots\ldots\ldots \text{Change in diameter of sphere}$$

$$e_V = 3e_C = 3\frac{\sigma_C}{E}(1 - \mu) \ldots\ldots\ldots\ldots \text{(iii) . Volumetric strain}$$

Two dimensional Stress System

Chapter-03

Introduction:

In earlier chapter we have always referred the stress in a plane which is right angles to the line of action of the force. (In case of tensile or compressive stress). Moreover we have consider at a time one type of stress acting in one direction only.

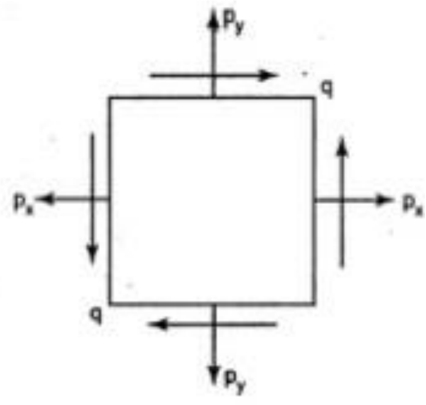


Fig. 3.2 2D Stress

In actual Engineering problems combination of stresses will act. The member may be subjected to direct stresses in different directions. The shear Stresses (direct or due to torsion) may also.

A beam is always under bending and shear. A shaft may be under torque, bending and direct forces. In this chapter we will see the effect of combined stresses.

In many problems two dimensional idealization is possible and the general stress system in such case is shown in fig.

❖ **Stresses on an Inclined plane (Oblique Plane):**

Though the state of stress at a point in a stressed body remains the same, the normal and shear stress components vary as the orientation of plane through that point changes. Under complex loading, a structural member may experience larger stresses on inclined planes then on the cross section.
The knowledge of maximum normal and shear stresses and their plane's orientation assumes significance from failure point of view. Hence, it is important to know how to transform the stress components from one set of coordinate axes to another set of co-ordinates axes that will contain the stresses of interest.

Oblique Plane:

It is defined as the plane which is at an angle of inclination $'\theta'$ with vertical axis of body under the action of direct stresses and shear stress. We have to determine the normal, tangential and resultant stress on inclined plane.

❖ Methods for determination of tangential stress, Normal stress and Resultant Stress on an inclined Plane
There following methods are used for determination of Stresses acting on oblique plane
i. Analytical Method
ii. Graphical Method

❖ Analytical Method for determination of stresses on an Oblique plane:
i. A body subjected to a direct stress in one plane
ii. A body subjected to direct stresses in two mutually perpendicular direction.
iii. Body subjected to direct stresses and shear stresses along two mutually perpendicular direction.

Derive an expression of normal and tangential stress on an oblique plane subjected to two mutually perpendicular direct stress. [2011, 2013, 2015, 2016]

Ans.:

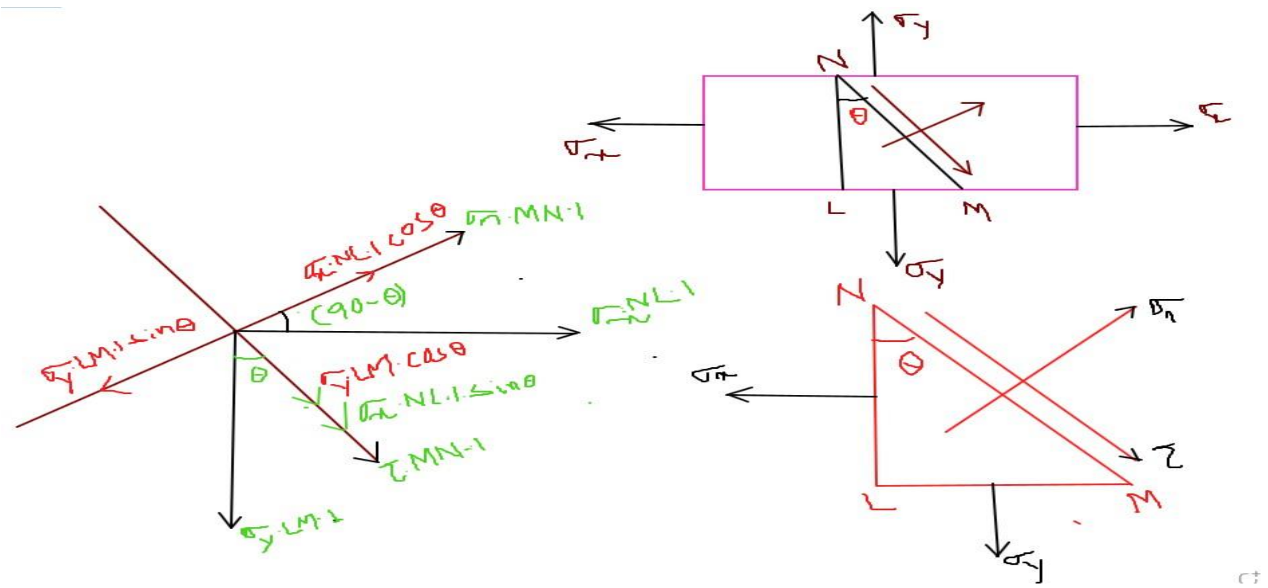
❖ Analytical Method for determination tangential stress, Normal stress and Resultant Stress on an inclined Plane, when body subjected direct stresses along two mutual perpendicular direction.

Let's consider a body ABCD of unit thickness '1'. The body is subjected to direct stresses σ_x & σ_y along X-axis and Y-axis respectively as shown in figure. Below let's consider a MN plane making angle of 'θ' with vertical plane LN of body.

Let's consider the triangle Δ LMN is subject to above mentioned stresses.
For determining of normal stress, shear stress acting on oblique plane considering equilibrium of body
$\sum \mathrm{H}(\text{Algebric Sum of Horizontal force}) = 0$, $\sum \mathrm{V}(\text{Algebric Sum of Vertical forces}) = 0$

Let

σ_x – Stress acting on body along X – direction
σ_y – Stress acting on body along Y – axis
θ – angle of inclination of inclined plane with vertical palne of body
σ_n – Normal Stress acting on inclined plane
τ – Shear Stress acting on inclined plane
σ_r – Resultant Stress acting on Inclined plane



I. Value of Normal Stress on Inclined Plane:

Considering the resolution of forces along Normal stress direction on oblique plane

$$\Rightarrow \sigma_n . MN . 1 = \sigma_x . LN \cos\theta . 1 + \sigma_y . LM..1 \sin\theta$$

$$\Rightarrow \sigma_n = \sigma_x . \left(\frac{LN}{MN}\right)\cos\theta + \sigma_y \left(\frac{LM}{MN}\right).\sin\theta$$

$$\Rightarrow \sigma_n = \sigma_x . \cos^2\theta + \sigma_y \sin^2\theta$$

$$\Rightarrow \sigma_n = \frac{2}{2}\sigma_x . \cos^2\theta + \frac{2}{2}\sigma_y \sin^2\theta$$

$$\Rightarrow \sigma_n = \frac{\sigma_x}{2}(1+\cos 2\theta) + \frac{\sigma_y}{2}(1-\cos 2\theta)$$

$$\Rightarrow \boldsymbol{\sigma_n} = \frac{\boldsymbol{\sigma_x + \sigma_y}}{\mathbf{2}} + \frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}} \mathbf{\cos 2\theta} \rightarrow \text{Normal stress acting on oblique plane}$$

II. Shear stress acting on oblique plane (τ):-

Considering the resolution of forces along tangential stress direction on oblique plane

$$\Rightarrow \tau . MN . 1 = \sigma_x . LN \sin\theta . 1 - \sigma_y . LM . \cos\theta . 1$$

$$\Rightarrow \tau = \sigma_x \left(\frac{LN}{MN}\right)\sin\theta - \sigma_y . \left(\frac{LM}{MN}\right). \cos\theta$$

$$\Rightarrow \tau = \sigma_x . \cos\theta . \sin\theta - \sigma_y . \cos\theta . \sin\theta$$

$$\Rightarrow \tau = \frac{\sigma_x}{2} 2\cos\theta . \sin\theta - \frac{\sigma_y}{2} . 2\cos\theta . \sin\theta$$

$$\Rightarrow \boldsymbol{\tau} = \frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}} \mathbf{\sin 2\theta} \rightarrow \text{Shear stress acting on oblique plane}$$

a. Maximum Normal Stress:

For the maximum value of normal stress

$$\cos 2\theta = 1 \Rightarrow 2\theta = \cos^{-1} 1 = 0°$$

Now putting value above $\cos 2\theta$ we have maximum normal stress

$$\sigma_{nMAX} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}.1 = 2\frac{\sigma_x}{2} = \sigma_x$$

$$\Rightarrow \boldsymbol{\sigma_{nMAX} = \sigma_x \dots\dots\dots\dots\dots \text{Major Stress acting on body}}$$

b. Minimum Normal Stress:
 For the minimum value of normal Stress

$$\cos 2\theta = -1 \Rightarrow 2\theta = \cos^{-1}(-1) = 180°$$

Now putting value above $\cos 2\theta$ we have maximum normal stress

$$\sigma_{nMIN} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}.(-1) = 2\frac{\sigma_y}{2} = \sigma_y$$

$$\Rightarrow \boldsymbol{\sigma_{nMIN} = \sigma_y \dots\dots\dots\dots\dots \text{Minor Stress acting on body}}$$

Major Plane: The plane upon which the magnitude of normal stress is maximum, called major plane
Minor Plane: The plane upon which the magnitude of normal stress is minimum, called minor plane

c. Maximum shear Stress:

For maximum value of shear stress value of

$$\sin 2\theta = 1 \Rightarrow 2\theta = \sin^{-1} 1 = 90°$$

$\Rightarrow \theta = 45°$
Now we have maximum shear stress

$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2}\sin 2\,\theta = \frac{\sigma_x - \sigma_y}{2}.1 = \frac{\sigma_x - \sigma_y}{2}$$

$$\Rightarrow \boldsymbol{\tau_{max} = \frac{\sigma_x - \sigma_y}{2} \dots\dots\dots\dots\dots\dots \text{Maximum shear stress on inclined plane}}$$

III. Resultant Stress :
 It is obtained by the expression

$$\boldsymbol{\sigma_r = \sqrt{\sigma_n^2 + \tau^2}}$$

Q. **At a point in a strained material the principal stresses are 100MPa and 50 MPa both tensile. Determine normal, tangential stresses at a section inclined at 60° with the axis of major principal stress.**

Ans.:

Given Data:

$\sigma_x = 100\text{ MPa}$
$\sigma_y = 50\text{ MPa}$
$\theta = 90° - 60° = 30°$
$\sigma_n, \tau = ?$

Value of Normal Stress:
We have

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$\frac{100 + 50}{2} + \frac{100 - 50}{2}\cos(2 \times 30) = 87.5\text{ MPa}$$

$$\boldsymbol{\sigma_n = 87.5\text{ MPa} \dots\dots\dots\dots\dots\dots\dots\dots \text{value of Normal stress}}$$

Value of Shear Stress:
We have

$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\,\theta$$

$$\frac{100-50}{2} \sin 2 \times 30 = 21.65 \text{ MPa}$$

$$\boldsymbol{\tau = 21.65\ \text{MPa} \ldots\ldots\ldots\ldots\ldots \text{value of Shear Stress}}$$

The stresses at a point of a machine component are 160 MPa and 50MPa, both tensile. Find the intensities of normal, shear and resultant stresses on a plane inclined at 60º with the axis of major tensile stress. Also find the magnitude of maximum shear stress in the component. [2013, 2017w]

Given data:

$\sigma_x = 160$ MPa

$\sigma_y = 50$ MPa

$\theta = 90° - 60° = 30°$

$\sigma_n, \tau = ?$

Value of Normal Stress:

We have

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

$$\frac{160+50}{2} + \frac{160-50}{2} \cos(2 \times 30) = 132.5 \text{ MPa}$$

$$\boldsymbol{\sigma_n = 132.5\ \text{MPa} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{value of Normal stress}}$$

Value of Shear Stress:

We have

$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\,\theta$$

$$\frac{160-50}{2} \sin 2 \times 30 = 47.63 \text{ MPa}$$

$$\boldsymbol{\tau = 47.63\ \text{MPa} \ldots\ldots\ldots\ldots\ldots \text{value of Shear Stress}}$$

Q. The stresses at a point of a machine component are 150MPa and 50MPa, both tensile. Find the intensities of normal, shear and resultant Stress on a plane inclined at an angle of 55° with the axis of minor tensile stress.

Ans.

Given Data:

$\sigma_x = 150$ MPa

$\sigma_y = 50$ MPa

$\theta = 55°$

$\sigma_n, \tau, \sigma_r = ?$

1. Value of Normal Stress:

We have

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

$$\frac{150+50}{2} + \frac{150-50}{2} \cos(2 \times 55) = 82.89 \text{ MPa}$$

$$\boldsymbol{\sigma_n = 82.89\ \text{MPa} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{value of Normal stress}}$$

2. Value of Shear Stress:

We have

$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\,\theta$$

$$\frac{150-50}{2}\sin 2\times 55 = 46.98\ \text{MPa}$$

$$\boldsymbol{\tau = 46.98\ MPa \ldots\ldots\ldots\ldots\ldots \text{value of Shear Stress}}$$

3. Value of Resultant Stress:

We have resultant Stress

$$\boldsymbol{\sigma_r = \sqrt{\sigma_n^2+\tau^2} = \sqrt{82.89^2+46.98^2} = 95.27\ MPa}$$

❖ **Analytical Method for determination tangential stress, Normal stress and Resultant Stress on an inclined Plane, when body subjected direct stresses with shear stress along two mutual perpendicular direction.**

Let's consider a body ABCD of unit thickness '1'. The body is subjected to direct stresses σ_x , σ_y and τ_{xy} along X-axis and Y-axis respectively as shown in figure. Below let's consider a MN plane making angle of 'θ' with vertical plane LN of body.

Let's consider the triangle Δ LMN is subject to above mentioned stresses.

For determining of normal stress, shear stress acting on oblique plane considering equilibrium of body

Let

σ_x − Stress acting on body along X − direction

σ_y − Stress acting on body along Y − axis

τ_{xy} − Complementary shear stres in body

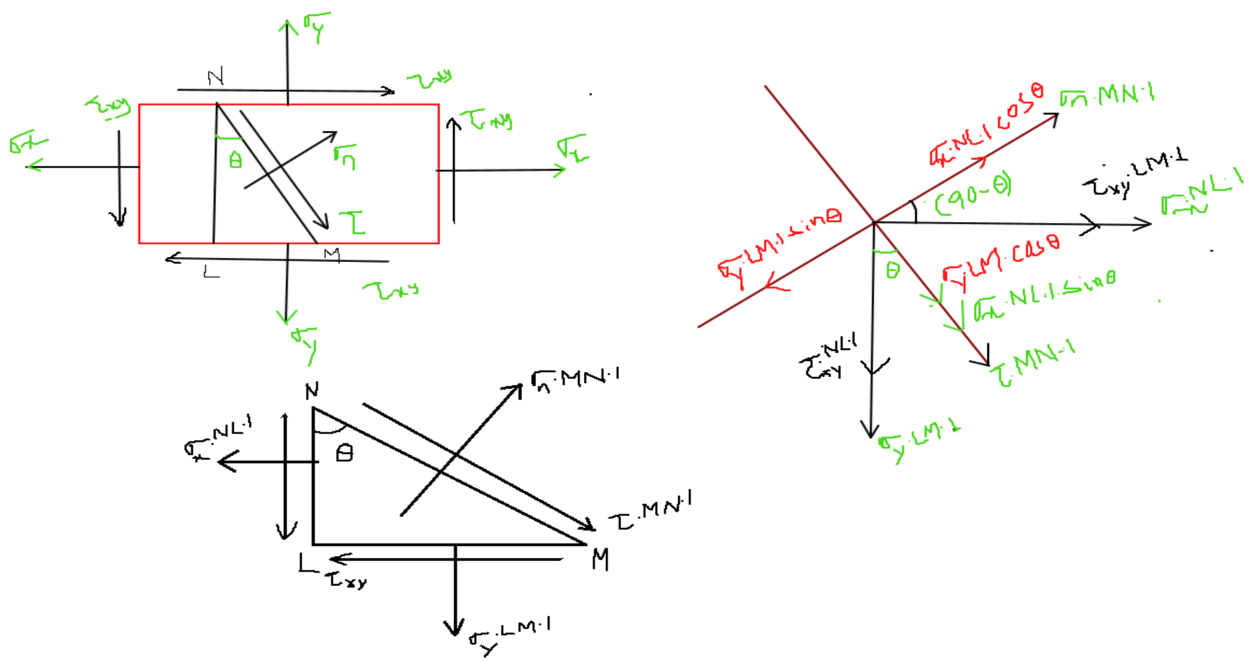
θ − angle of inclination of inclined plane with vertical palne of body

σ_n − Normal Stress acting on inclined plane

τ − Shear Stress acting on inclined plane

σ_r − Resultant Stress acting on Inclined plane

❖ **Complementary Shear Stress:**



I. Value of Normal Stress on Inclined Plane:

Considering the resolution of forces along Normal stress direction on oblique plane

$$\sigma_n.MN.1 = \sigma_x.NL.1.\cos\theta + \tau_{xy}.LM.1.\cos\theta + \tau_{xy}.NL.1.\sin\theta + \sigma_y.LM.1.\sin\theta$$

$$\sigma_n = \sigma_x.\frac{NL}{MN}.\cos\theta + \tau_{xy}.\frac{LM}{MN}.\cos\theta + \tau_{xy}.\frac{NL}{MN}.\sin\theta + \sigma_y.\frac{LM}{MN}.\sin\theta$$

$$\sigma_n = \sigma_x.\cos\theta.\cos\theta + \tau_{xy}.\sin\theta.\cos\theta + \tau_{xy}.\cos\theta.\sin\theta + \sigma_y.\sin\theta.\sin\theta$$

$$\sigma_n = (\sigma_x.\cos\theta.\cos\theta + +\sigma_y.\sin\theta.\sin\theta) + (\tau_{xy}.\sin\theta.\cos\theta + \tau_{xy}.\cos\theta.\sin\theta)$$

$$\sigma_n = (\sigma_x.\cos^2\theta + +\sigma_y.\sin^2\theta) + (2\tau_{xy}.\sin\theta.\cos\theta)$$

$$\boldsymbol{\sigma_n = (\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta) + \tau_{xy}.\sin 2\theta \ldots\ldots\ldots.\text{Normal stress acting on inclined plan}}$$

Value of shear stress acting on inclined plane:

Resolving forces along tangential direction we have

$$\tau.MN.1 + \tau_{xy}.NL.1.\cos\theta + \sigma_y.LM.1.\cos\theta = \sigma_x.NL.1.\sin\theta + \tau_{xy}.LM.1.\sin\theta$$

$$\tau = \sigma_x.\frac{NL}{MN}.\sin\theta + \tau_{xy}.\frac{LM}{MN}.\sin\theta - \tau_{xy}.\frac{NL}{MN}.\cos\theta - \sigma_y.\frac{LM}{MN}.\cos\theta$$

$$\tau = \sigma_x.\cos\theta.\sin\theta + \tau_{xy}.\sin\theta.\sin\theta - \tau_{xy}.\cos\theta.\cos\theta - \sigma_y.\sin\theta.\cos\theta$$

$$\tau = (\sigma_x.\cos\theta.\sin\theta - \sigma_y.\sin\theta.\cos\theta) + (\tau_{xy}.\sin\theta.\sin\theta - \tau_{xy}.\cos\theta.\cos\theta)$$

$$\tau = 2\,(\frac{\sigma_x - \sigma_y\sin\theta.\cos\theta}{2}) - (\tau_{xy}.\cos^2\theta - \tau_{xy}.\sin^2\theta)$$

$$\boldsymbol{\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \ldots\ldots\ldots\ldots\ldots.\text{Shear stress on oblique plane}}$$

- **Principal plane:**

It is defined as the plane upon which the value of shear stress is Zero.

- **Location Principal plane :**

We know that the value of shear stress is zero on principal plane
Now we have Shear stress

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta = 0$$

$$\Rightarrow \frac{\sigma_x - \sigma_y}{2}\sin 2\theta = \tau_{xy}\cos 2\theta$$

$$\Rightarrow \tan 2\theta = \frac{\tau_{xy}}{(\frac{\sigma_x - \sigma_y}{2})}$$

$$\boldsymbol{\Rightarrow 2\theta = \tan^{-1}\frac{\tau_{xy}}{(\frac{\sigma_x - \sigma_y}{2})} \ldots\ldots\ldots\ldots.\text{Location of Principal plane}}$$

$$\text{now } \sin 2\theta = \frac{\tau_{xy}}{\sqrt{((\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2)}}$$

$$\cos 2\theta = \frac{(\frac{\sigma_x - \sigma_y}{2})}{\sqrt{((\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2)}}$$

- **Calculation of Pricipal Stresses :**

Principal Stress:

The principal plane is subjected to normal stresses known as principal stresses. Since we know that there is no shear stress on principal plane hence the normal stresses are acting on it. There are two types of principal stresses according to the maximum and minimum value of normal stress.

i. Maximum Principal stress
ii. Minimum Principal stress

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}.\sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\sqrt{\left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)}} + \tau_{xy}.\frac{\tau_{xy}}{\sqrt{\left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)}}$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2}{\sqrt{\left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)}} + \frac{\tau_{xy}^2}{\sqrt{\left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)}}$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}{\sqrt{\left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)}}$$

$$\boldsymbol{\sigma_{nmax}} = \frac{\boldsymbol{\sigma_x + \sigma_y}}{\mathbf{2}} + \sqrt{\left(\left(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}\right)^2 + \boldsymbol{\tau_{xy}^2}\right)} \ldots\ldots\ldots\ldots\ldots \textbf{Maximum Principal Stress}$$

$$\boldsymbol{\sigma_{nmin}} = \frac{\boldsymbol{\sigma_x + \sigma_y}}{\mathbf{2}} - \sqrt{\left(\left(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}\right)^2 + \boldsymbol{\tau_{xy}^2}\right)} \ldots\ldots\ldots\ldots\ldots \textbf{Minimum Principal Stress}$$

Maximum Shear stress on Principal Plane:

We have value of shear Stress

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

Now taking derivative of shear stress with respect to angle of Inclination ‘θ’and should be equal to zero.

$$\tau_{Max} = \frac{d\tau}{d\theta} = \frac{d\left(\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right)}{d\theta} = 0$$

$$\Rightarrow \tau_{Max} = \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta = 0$$

$$\Rightarrow \tau_{Max} = \frac{\sigma_x - \sigma_y}{2} \cdot \frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\sqrt{\left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)}} + \tau_{xy} \cdot \frac{\tau_{xy}}{\sqrt{\left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)}}$$

$$\Rightarrow \tau_{Max} = \frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}{\sqrt{\left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)}} = \sqrt{\left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)}$$

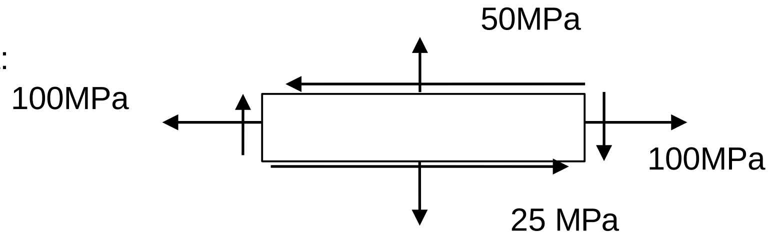
$$\Rightarrow \tau_{Max} = \sqrt{\left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)}$$

$$\boldsymbol{\tau_{Max}} = \sqrt{\left(\left(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}\right)^{\mathbf{2}} + \boldsymbol{\tau_{xy}^2}\right)} \text{ .... Maximum Shear Stress}$$

Q. A point in a material is subjected to a stress as shown below. Calculate
i) Principal Stresses
ii) Maximum shear stress and also the plane along which it.

Ans:

Given Data:



Given Data:

$\sigma_x = 100$ MPa

$\sigma_y = 50$ MPa

$\tau_{xy} = 25$ Mpa

$\sigma_{nMax}, \sigma_{nMin} \tau_{max} = ?$

Maximum Principal Stress:

We have

$$\boldsymbol{\sigma_{nmax}} = \frac{\boldsymbol{\sigma_x + \sigma_y}}{\mathbf{2}} + \sqrt{\left(\left(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}\right)^{\mathbf{2}} + \boldsymbol{\tau_{xy}^2}\right)}$$

$$= \frac{\mathbf{100 + 50}}{\mathbf{2}} + \sqrt{\left(\left(\frac{\mathbf{100 - 50}}{\mathbf{2}}\right)^{\mathbf{2}} + \mathbf{25^2}\right)}$$

Minimum Principal Stress:

We have

$$\boldsymbol{\sigma_{nmin}} = \frac{\boldsymbol{\sigma_x + \sigma_y}}{\mathbf{2}} - \sqrt{\left(\left(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}\right)^{\mathbf{2}} + \boldsymbol{\tau_{xy}^2}\right)}$$

$$= \frac{\mathbf{100 + 50}}{\mathbf{2}} - \sqrt{\left(\left(\frac{\mathbf{100 - 50}}{\mathbf{2}}\right)^{\mathbf{2}} + \mathbf{25^2}\right)}$$

Maximum Shear Stress:

$$\tau_{Max} = \sqrt{\left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)}$$

$$= \sqrt{\left(\left(\frac{100 - 50}{2}\right)^2 + 25^2\right)} =$$

❖ **Graphical Method for determination of Normal Stress, Shear Stress and Resultant stress on oblique plane of body**

It is done by drawing Mohr's Circle of Stresses. The construction of Mohr's Circle of stresses as well as determination of normal, shear and resultant stresses is very easier than the analytical Method. Moreover there is a little chance of committing any error in this method.

Mohr's Circle:

Mohr's circle, invented by Christian Otto Mohr, is a two-dimensional graphical representation of the stresses acting on oblique plane

i. Mohr's circle is often used in calculations relating to mechanical engineering for materials' strength, geotechnical engineering for strength of soils, and structural engineering for strength of built structures.
ii. It is also used for calculating stresses in many planes by reducing them to vertical and horizontal components. These are called principal planes in which principal stresses are calculated; Mohr's circle can also be used to find the principal planes and the principal stresses in a graphical representation.

Mohr's Circle of stresses drawn for following cases

1. A body subjected to direct stresses(Like and Unlike) in two mutually perpendicular direction
2. A body subjected to direct stresses in two mutually perpendicular direction accompanied by a simple shear

Mohr-circle-space sign convention

i. In the Mohr-circle-space sign convention, normal stresses have the same sign as normal stresses in the physical-space sign convention: positive normal stresses act outward to the plane of action, and negative normal stresses act inward to the plane of action
ii. In the Mohr-circle-space sign convention, This way, the shear stress component is positive in the Mohr-circle space, and the shear stress component is negative in the Mohr-circle space
iii. Two options exist for drawing the Mohr-circle space, which produce a mathematically correct Mohr circle:

1. Positive shear stresses are plotted upward
2. Positive shear stresses are plotted downward

Plotting positive shear stresses upward makes the angle 2θ on the Mohr circle have a positive rotation clockwise, which is opposite to the physical space convention. That is why some authors [3] prefer plotting positive shear stresses downward, which makes the angle 2θ on the Mohr circle have a positive rotation counter clockwise, similar to the physical space convention for shear stresses

❖ Graphical Method for determination tangential stress, Normal stress and Resultant Stress on an inclined Plane, when body subjected direct stresses along two mutual perpendicular direction

1. **Mohr's Circle for body subjected to direct stresses (Like) in two mutually perpendicular direction**

Let's consider a body ABCD of unit thickness '1'. The body is subjected to direct stresses σ_x & σ_y along X-axis and Y-axis respectively as shown in figure. Let's consider a MN plane making angle of 'θ' with vertical plane LN of body.

Let's consider the triangle Δ LMN is subject to above mentioned stresses.

For determining of normal stress, shear stress acting on oblique plane considering equilibrium of body

Let

σ_x – Stress acting on body along X – direction

σ_y – Stress acting on body along Y – axis

θ – angle of inclination of inclined plane with vertical palne of body

σ_n – Normal Stress acting on inclined plane

τ – Shear Stress acting on inclined plane

σ_r – Resultant Stress acting on Inclined plane

- **Procedure:**

a. At first choose a suitable scale ($1\ \text{cm} \sim \frac{\text{N}}{\text{mm}^2}$) find equivalent magnitude of σ_x, σ_y in centimetre

b. Represent the point 'O', Draw line OL for direct stress σ_x and line OM for direct stress σ_y lie on same direction as OM

c. Bisect the line ML to find point 'N'

d. Draw the circle by taking radius MN or LN

e. Now measure value of '2θ' and join the points N & P to find 'NP'

f. Drop perpendicular $PQ \perp NL$

g. Join the line from point 'O' to 'P'

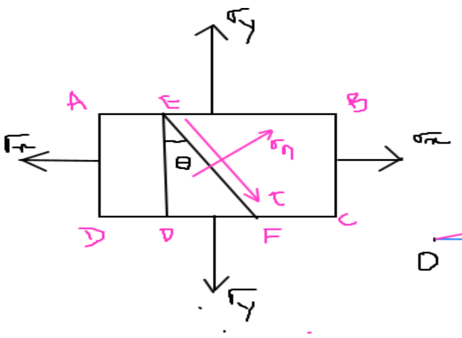
h. From Right angle Triangle $\Delta\ OPQ$ the magnitude of each side gives the stresses acting

i. Measurement of Normal Stress, Tangential Stress and tangential Stress

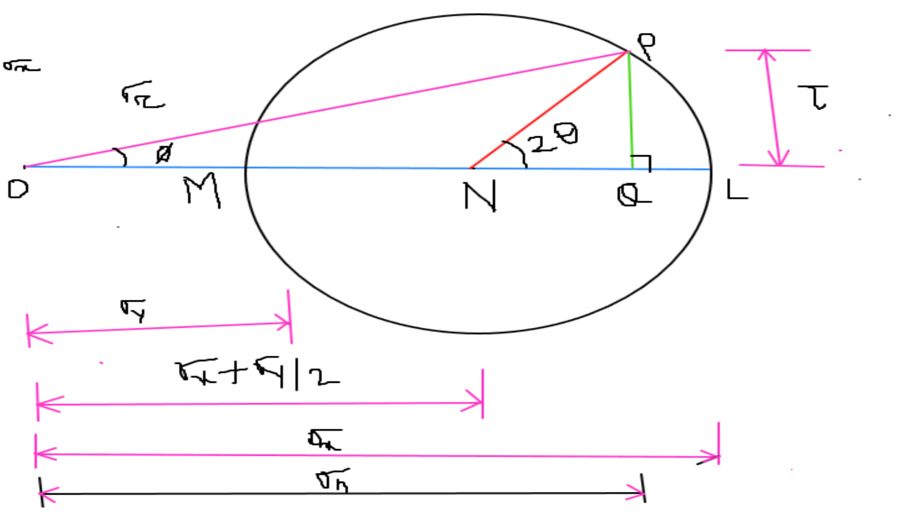
magnitude of $PQ = \tau$,

magnitude of $OQ = \sigma_n$

magnitude of $OP = \sigma_r$



MOHR'S CIRCLE FOR LIKE STRESSES



Verification of Mohr's Circle:

a. Value of Normal stress

We have from Right angle Triangle $\Delta\ OPQ$

$$\mathbf{OQ = ON + NQ}$$

$$OQ = (OM + MN) + NP\ \cos 2\theta$$

$$OQ = (OM + \frac{ML}{2}) + NL\ \cos 2\theta$$

$$OQ = (\sigma_y + \frac{\sigma_x - \sigma_y}{2}) + \frac{\sigma_x - \sigma_y}{2}\ \cos 2\theta$$

$$OQ = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta = \boldsymbol{\sigma_n}$$

$$\left(\frac{\boldsymbol{\sigma_x + \sigma_y}}{\boldsymbol{2}}\right) + \frac{\boldsymbol{\sigma_x - \sigma_y}}{\boldsymbol{2}} \,\mathbf{cos\, 2\theta} = \boldsymbol{\sigma_n} \ldots\ldots\ldots\ldots\ldots \textbf{value of Normal stress as in analytical method}$$

b. Value of Shear stress:

We have from Right angle Triangle $\Delta\, OPQ$

$$PQ = NP\, \sin 2\theta = NL \sin 2\theta$$

$$\Rightarrow \frac{\boldsymbol{\sigma_x - \sigma_y}}{\boldsymbol{2}} .\,\mathbf{sin\, 2\theta} = \boldsymbol{\tau} \ldots\ldots\ldots\ldots.. \textbf{value of shear stress as in Analytical Method}$$

c. Value of Resultant Stress:

We have from Right angle Triangle $\Delta\, OPQ$

$$OP = \sqrt{OQ^2 + PQ^2}$$

$$\boldsymbol{\sigma_r} = \sqrt{\boldsymbol{\sigma_n}^2 + \boldsymbol{\tau}^2} \ldots\ldots\ldots \textbf{value of resultant stress as in Analytic Method}$$

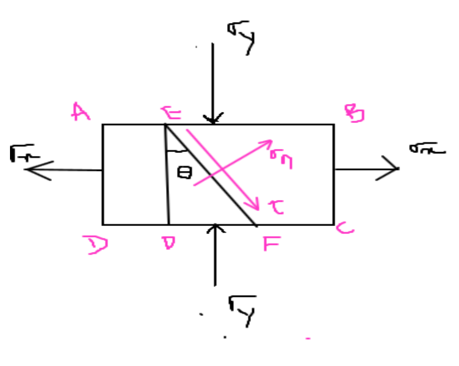
3. **Mohr's Circle for body subjected to direct stresses (Un Like) in two mutually perpendicular direction.**

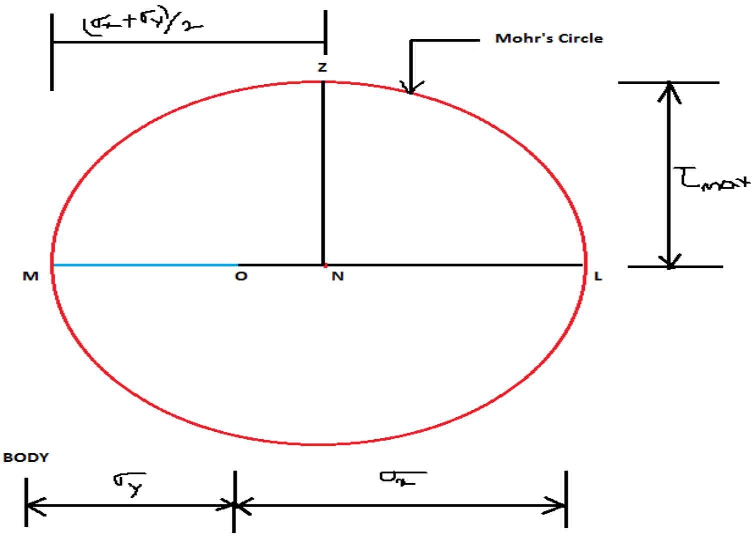
Whenever a body is subjected to two dimensional stresses such as one is +ve as tensile along X-direction and another is compressive –ve. along Y- direction. Then the body is stressed with unlike stresses.

a. At first choose a suitable scale ($1\ cm \sim \frac{N}{mm^2}$) find equivalent magnitude of σ_x, σ_y in centimetre
b. Represent the point 'O', Draw line OL for direct stress σ_x and line OM for direct stress σ_y lie in a opposite direction as OM
c. Bisect the line ML to find point 'N'
d. Draw the circle by taking radius MN or LN
e. Now measure value of $'2\theta'$ and join the points N & P to find 'NP'
f. Drop perpendicular $PQ \perp NL$
g. Join the line from point 'O' to 'P'
h. From Right angle Triangle $\Delta\, OPQ$ the magnitude of each side gives the stresses acting
i. Measurement of Normal Stress, Tangential Stress and tangential Stress
   magnitude of $PQ = \tau$,

$$\text{magnitude of } OQ = \sigma_n$$
$$\text{magnitude of } OP = \sigma_r$$

j. Draw line NZ which represents Maximum Shear stress





MOHR'S CIRCLE FOR UNLIKE STRESSES ACTING BODY

4. Mohr's Circle for Principal Stresses on body subjected to direct stresses and simple shear stress in two mutually perpendicular direction.

Let's consider a body ABCD of unit thickness '1'. The body is subjected to direct stresses σ_x , σ_y and τ_{xy} along X-axis and Y-axis respectively as shown in figure. Below let's consider a MN plane making angle of 'θ' with vertical plane LN of body.

Let's consider the triangle Δ LMN is subject to above mentioned stresses.

For determining of normal stress, shear stress acting on oblique plane considering equilibrium of body

Let

σ_x – Stress acting on body along X – direction

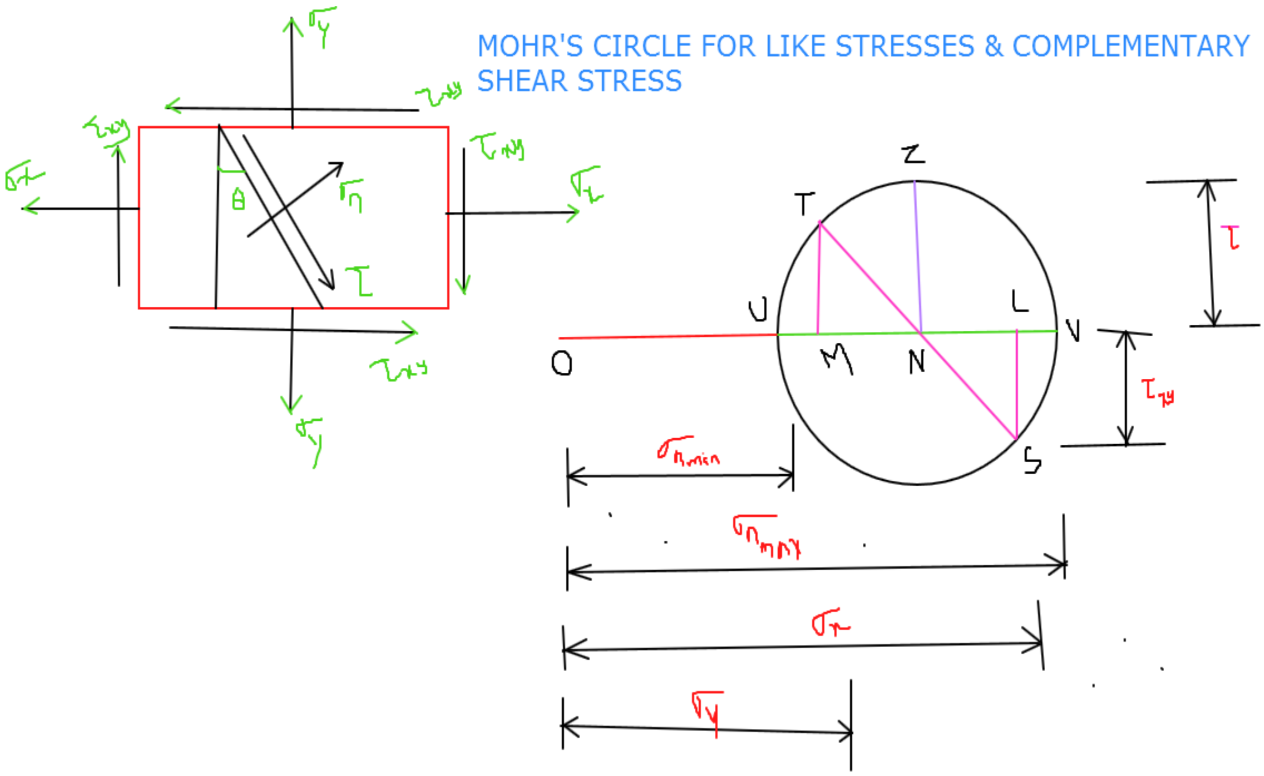
σ_y – Stress acting on body along Y – axis

τ_{xy} – Complementary shear stres in body

θ – angle of inclination of inclined plane with vertical palne of body

a. At first choose a suitable scale ($1\ \mathrm{cm} \sim \frac{N}{mm^2}$) find equivalent magnitude of σ_x , σ_y in centimetre
b. Represent the point 'O', Draw line OL for direct stress $'\sigma_x'$ and line OM for direct stress $'\sigma_y'$ lie in same direction as OM
c. Drop perpendicular LT and MS with line OM for represents complementary Shear Stress (τ_{xy}) (Counter clockwise /anticlockwise)
d. Draw the line ST from point 'S' to 'T' which intersecting the line OM at point 'N'
e. Draw the Circle at point 'N' with radius NT or SN
f. The circle crossing the line OM at points 'U' and 'V'
g. Now drop perpendicular NZ on line OM from point 'N'
h. The angle subtend by line ST with ML is the Angle of Obliquity 2θ
i. The value of Maximum Shear stress

$$\text{Magnitude of } \mathbf{NZ} = \boldsymbol{\tau}_{\mathbf{Max}}$$
$$\text{magitude of } \mathbf{OU} = \boldsymbol{\sigma}_{\mathbf{n\ min}}$$
$$\text{magnitude } \mathbf{OV} = \boldsymbol{\sigma}_{\mathbf{n\ max}}$$



- **Verification Mohr's Circle with Analytical Method for determination for Principal Stresses:**

Maximum Principal Stress:

Form Mohr's Circle value Maximum Principal Stress

$OV = ON + NV$

$$= (OM + MN) + NT$$

$$= (OM + \frac{OL - OM}{2}) + (\sqrt{NL^2 + TL^2})$$

$$= (OM + \frac{OL - OM}{2}) + (\sqrt{(OL - OM)^2 + TL^2})$$

$$= (\sigma_y + \frac{\sigma_x - \sigma_y}{2}) + (\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}{}^2})$$

$$\boldsymbol{\sigma_{n\,max}} = (\boldsymbol{\sigma_y} + \frac{\boldsymbol{\sigma_x} - \boldsymbol{\sigma_y}}{\mathbf{2}}) + (\sqrt{(\frac{\boldsymbol{\sigma_x} - \boldsymbol{\sigma_y}}{\mathbf{2}})^2 + \boldsymbol{\tau_{xy}}^2}) \dots\dots\dots\dots\dots\dots \textbf{Maximum Principal Stress}$$

Minimum Principal Stress:

From above diagram of Mohr's Circle Minimum principal Stress

$OU = ON - UN$

$= (OM + MN) - NS$

$= (OM + \frac{OL - OM}{2}) - NT$

$$= (OM + \frac{OL - OM}{2}) - (\sqrt{NL^2 + TL^2})$$

$= (OM + \frac{OL - OM}{2}) - (\sqrt{(OL - OM)^2 + TL^2})$

$$= (\sigma_y + \frac{\sigma_x - \sigma_y}{2}) + (\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2})$$

$$\boldsymbol{\sigma_{n\,Min}} = (\boldsymbol{\sigma_y} + \frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}) + (\sqrt{(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}})^2 + \boldsymbol{\tau_{xy}^2}}) \ldots\ldots\ldots\ldots\ldots\ldots \textbf{Minimum Principal Stress}$$

Maximum Shear Stress:

Maximum Shear Stress from Mohr's Circle is given by

$$NZ = NT$$

$$= \sqrt{NL^2 + TL^2}$$

$$= \sqrt{(OL - OM)^2 + TL^2}$$

$$\tau_{Max} = \sqrt{(\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}})^2 + \boldsymbol{\tau_{xy}^2}} \ldots\ldots\ldots\ldots\ldots\ldots . \textbf{Maximum Shear Stress}$$

Location of Principal Plane:

Form Mohr's circle the location of principal is given by

$$\tan 2\theta = \frac{TL}{NL} = \frac{\boldsymbol{\tau_{xy}}}{\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}}$$

$$2\theta = \tan^{-1} \frac{\boldsymbol{\tau_{xy}}}{\frac{\boldsymbol{\sigma_x - \sigma_y}}{\mathbf{2}}} \text{...............location of principal Plane}$$

Chapter-4
Shear force and Bending Moment

- **Beam:**
- A beam is a structural element that primarily resists loads applied laterally to the beam's axis. Its mode of deflection is primarily by bending.
- The loads applied to the beam result in reaction forces at the beam's support points.
- The total effect of all the forces acting on the beam is to produce shear forces and bending moments within the beam, that in turn induce internal stresses, strains and deflections of the beam.
- Beams are characterized by their manner of support, profile (shape of cross-section), equilibrium conditions, length, and their material.

- **Types of Beam**

There are following types of beam

1. Cantilever Beam
2. Simple Supported Beam
3. Continuous Beam
4. Fixed Beam
5. Overhanging Beam
6.

Cantilever Beam:-

A beam having one end fixed and other end is free, on the free end of beam loads are applied is called cantilever beam.

Fixed Beam:-

If the two ends of beam are fixed then is known as fixed beam

Simple supported beam:-

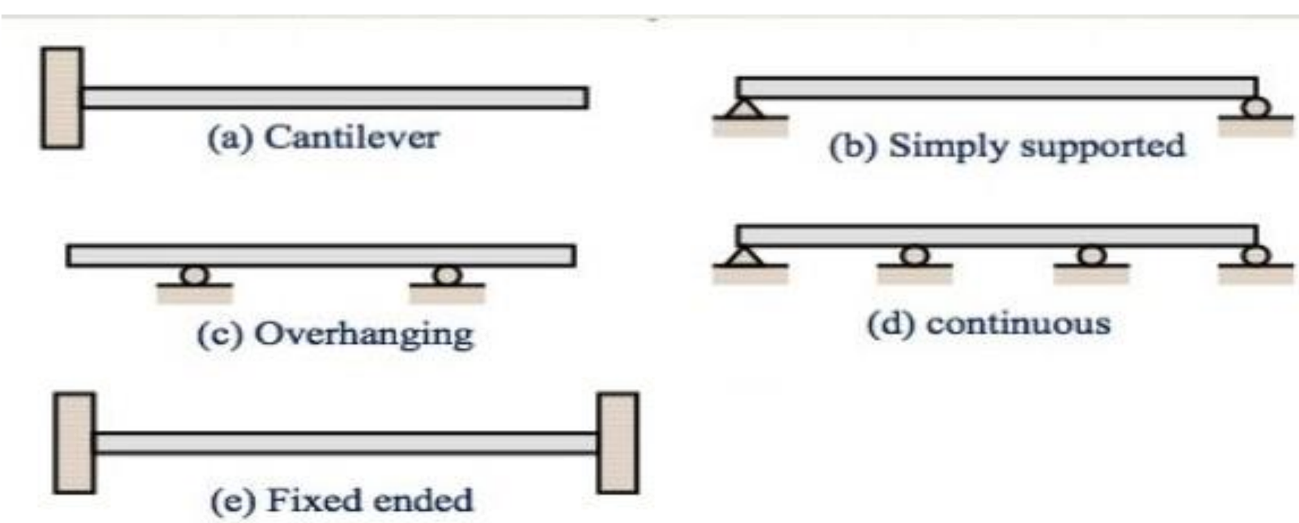
Whenever the beam is resting upon the two supports at its both ends is known as simple supported beam

Continuous beam:

Whenever more than two no's of supports are given to the beam throughout its length is called continuous beam

Overhang Beam:-

If the certain portion of beam is extended beyond the supports as shown in diagram, is called overhang beam



Type of load on Beam:

There are three types of load

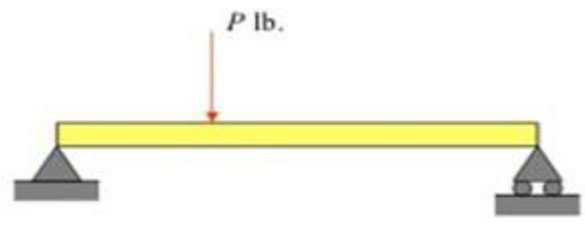
1. Point Load
2. Uniformly Distributed load
3. Uniform varying Load

Point Load:

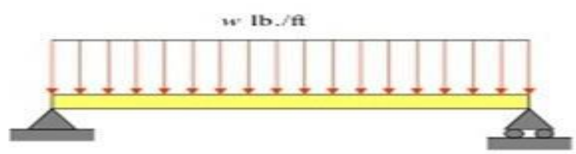
It is also the concentrated load which is acting at a point on beam.

Uniformly Distributed Load:

- It is spread over the beam in such a way that the rate of loading is uniform along the length (i.e. each unit length is loaded to the same rate) as shown in diagram.
- It is expressed as shortly UDL in unit N over Meter run
- For solving the numerical problem the total uniformly distributed load is converted into a point load ,acting at the center of uniformly distributed load

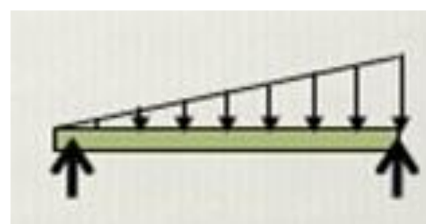


Concentrated/point load



UDL: Uniformly distributed load

Uniformly Varying Load:



- It is load which is spread over a beam in such a manner that rate of loading varies from point to point along the beam as shown in diagram.
- In this type load is zero at one end and increase uniformly to the other end. Is called as Triangular load.
- For solving problem the total load is equal to the area of triangle and total load is assumed to be acting at C.G. of triangle.

Shear Force:

It is the algebraic Sum of all vertical forces acting either to the left or right side of a section of beam

Some important hints to be noted

1. Consider left part or right part of the section
2. Add the forces normal to the member on one of the parts
3. During Right part Selection : Downward forces are Positive, Upward Forces are Negative
4. During Left part Selection : Downward forces are negative, Upward Forces are Positive

- **Shear Force Diagram:**

  It is the diagrammatic representation for value of shear forces at various section of the member

- **Bending Moment:**

It is the algebraic sum of moments of all vertical forces acting to the right or left part of section of a beam

Some important hints to be noted

1. Consider left part or right part of the section
2. Remove all restraints and all forces of part selected

3. Calculate the effect of all forces at each point of beam
4. Take bending moment produced by Downward forces is Negative
5. Take bending moment produced by Upward forces is Positive

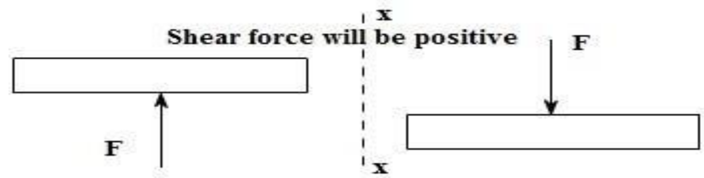
Bending Moment Diagram:

It is the diagrammatic representation for value of Bending Moment produced by shear forces at various section of the member

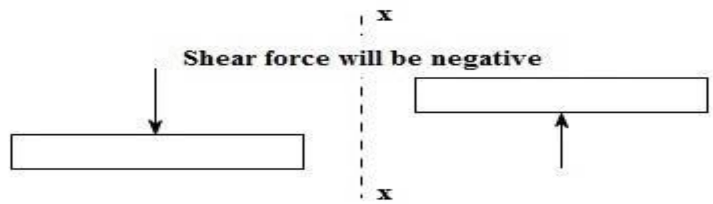
Sign Convention for Shear Force & Bending Moment :-

During Shear Force Calculation for a beam

- For Choosing Right Part of Section of Beam
  1. Down Forces are considered as Positive (+ VE)
  2. Upward Forces are considered as negative (-VE)



- For Choosing Left Part of Section of Beam
  1. Down Forces are considered as Negative (- VE)
  2. Upward Forces are considered as Positive (+ VE)



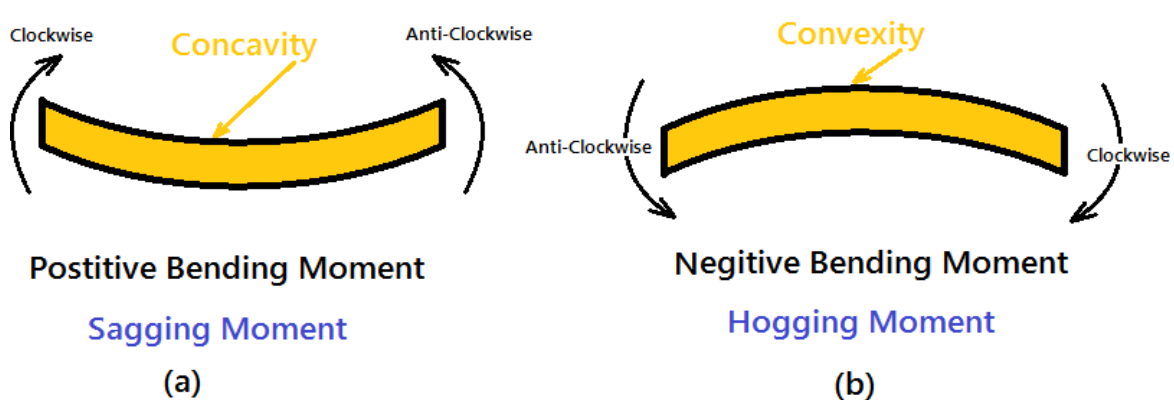
Bending Moment is of two types

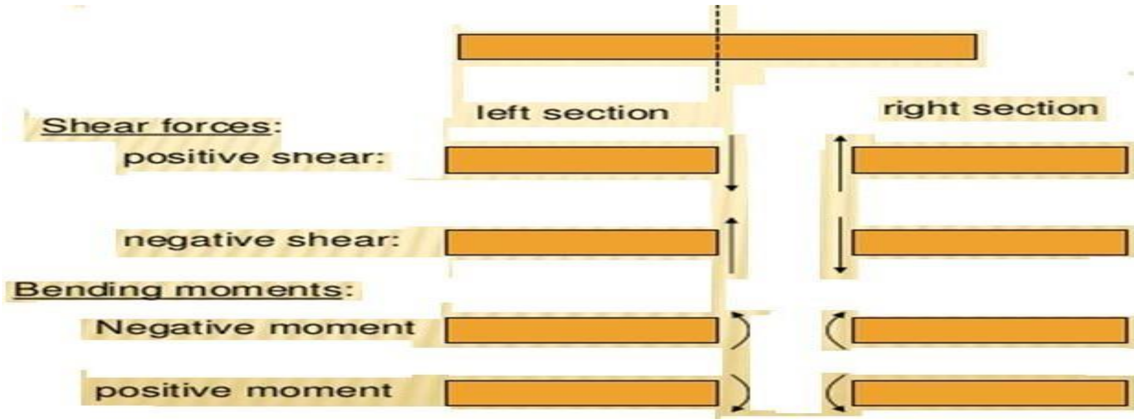
1. Hogging Bending Moment:

- It produces convexity of Beam at a section below its center line
- It is produced by the downward vertical forces acting on beam
- It is the negative Bending Moment

2. Sagging Bending Moment:

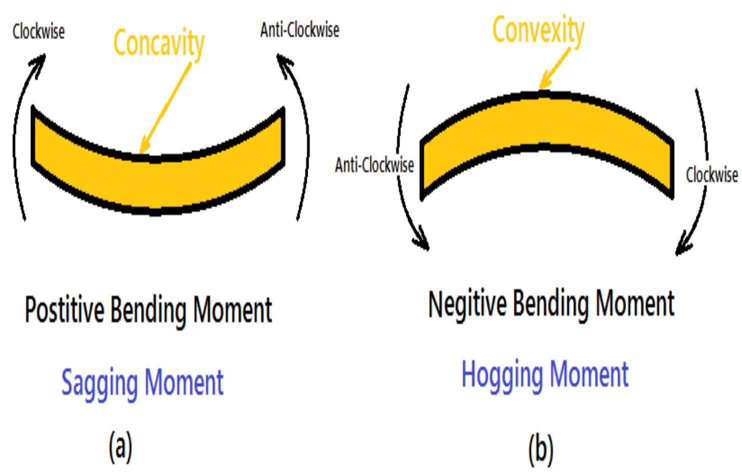
- It may Produce Concavity of beam at a section above its center line
- It is produced by the upward vertical forces acting on beam
- It is the Positive Bending Moment





Difference between Sagging Bending Moment and Hogging Bending Moment:

- Hogging Bending Moment:
    - It produces convexity of Beam at a section below its center line
    - It is produced by the downward vertical forces acting on beam
    - It is the negative Bending Moment
- Sagging Bending Moment:
    - It may Produce Concavity of beam at a section above its center line
    - It is produced by the upward vertical forces acting on beam
    - It is the Positive Bending Moment
    -



- **Calculation of Shear Force & Bending Moment Diagram for Cantilever beam**

Consider a cantilever beam as shown in diagram

Consider a section X-X at a distance of 'x' from the free end

$\text{S. F. at } X - X \text{ section} = S_X = +W,$

$\text{Bending moment at } X - X \text{ Section} = M_X = -W.x$

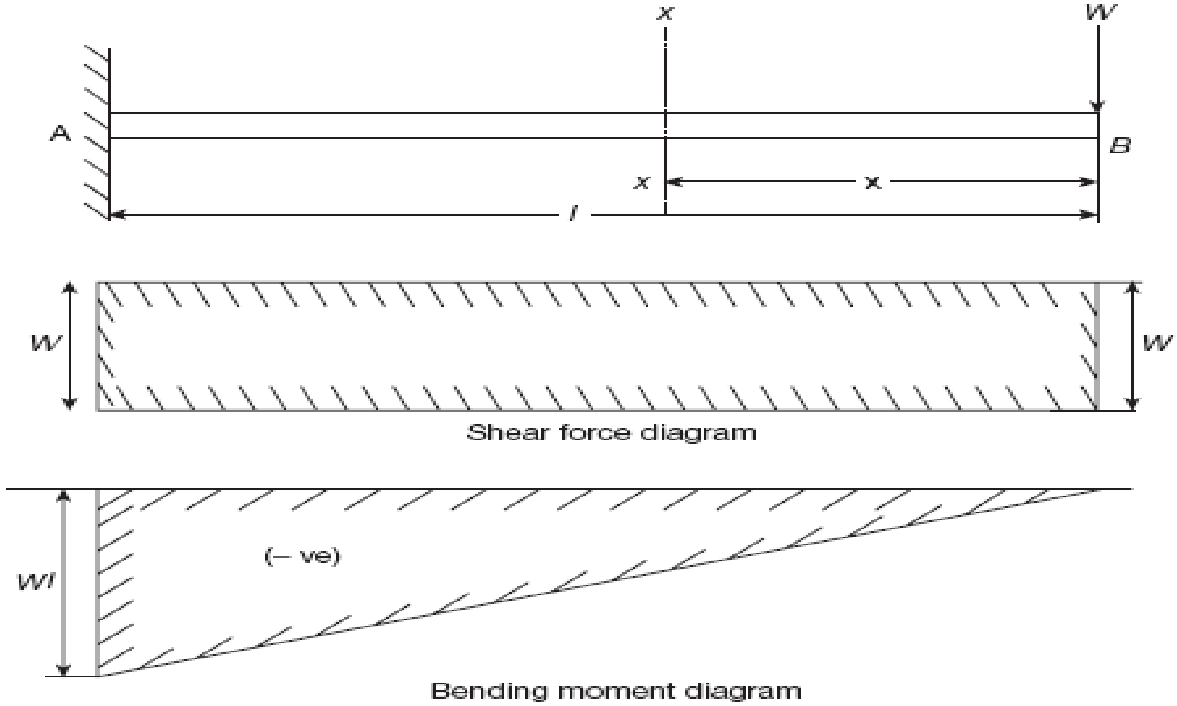
Shear force is constant at all the sections of member between A to B

$\text{B. M at B point}, x = 0, \ M_B = -W.0 = 0$

$\text{B. M at A point}, x = l, \ M_A = -W.l = -Wl$

Maximum Bending Moment for the Beam

$$\mathbf{M_{max} = Wl}$$



Cantilever Beam of Length 'l' carrying Uniformly Distributed Load (UDL)

Considering a cantilever beam as shown in diagram carrying a UDL 'W' per meter Run over the whole Span of Beam

$\mathbf{Consider\ a\ section\ X - X\ at\ a\ distance\ of\ 'x'\ from\ the\ free\ end}$

$\mathbf{S.F.\ at\ X - X\ section = S_X = \ + W.x,}$

$\mathbf{Bending\ moment\ at\ X - X\ Section}$

$\mathbf{M_X = -W.x.(x/2)}$

$\mathbf{Shear\ Force\ at\ B\ ,\ x = 0,\ S_B = 0}$

$\mathbf{Shear\ Force\ at\ A\ ,\ x =,\ S_B = +W.x}$

$\mathbf{B.M\ at\ A\ point, x = 0,}$

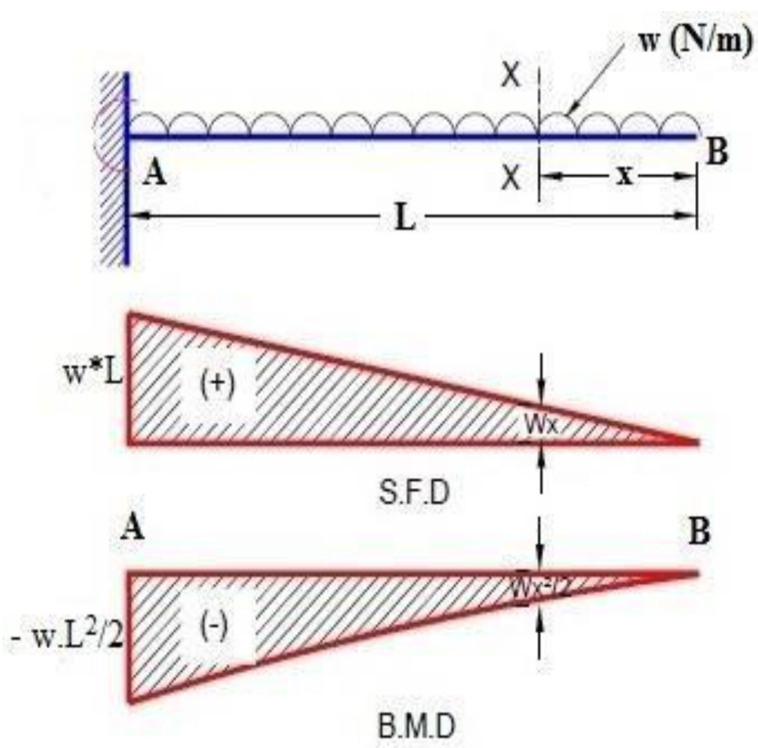
$\mathbf{M_A = -W.x.(x/2) = -W.0.(0/2) = 0}$

$\mathbf{B.M\ at\ B\ point, x = L, M_B = -W.x.(x/2)}$

$\mathbf{M_B = -W.L.(L/2) = WL^2/2}$

Maximum Bending Moment for Cantilever Beam

$$\mathbf{M_{max} = \frac{WL^2}{2}}$$



❖ **Cantilever Beam of Length 'l' carrying Uniformly Distributed Load (UDL)& Point load at Free end.**

Consider a section X − X at a distance of 'x' from the free end

$\mathbf{S.F.\ at\ X - X\ section = S_X = +2.x + 3\ KN,}$

Bending moment at X − X Section

$\mathbf{M_X = -2.x.(x/2) - 3 \times x}$

$\mathbf{Shear\ Force\ at\ B,\ x = 0,}$

$\mathbf{S_B = +2 \times 0 + 3\ KN}$

$\mathbf{S_B = +3\ KN}$

$\mathbf{Shear\ Force\ at\ A,\ x = 2,\ S_A = +2 \times 2 + 3\ KN}$

$\mathbf{S_A = +7\ KN}$

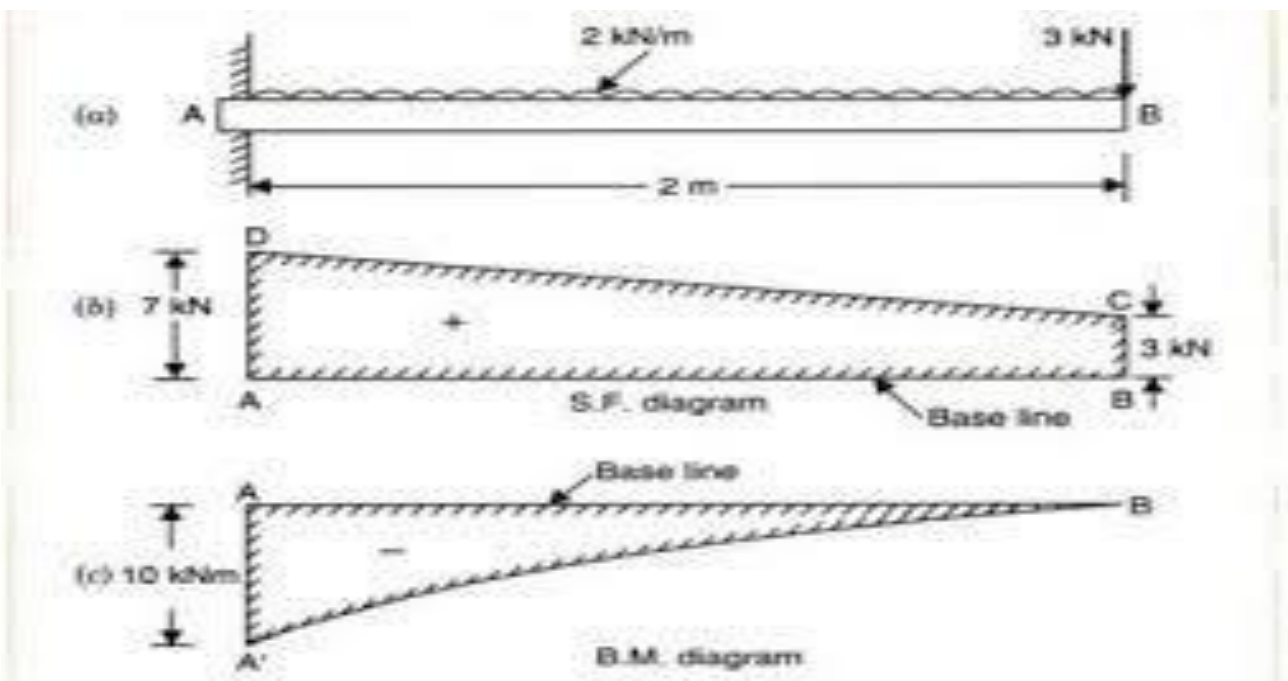
$\mathbf{B.M\ at\ B\ point, x = 0,}$

$\mathbf{M_B = -2.x.(x/2) - 3x \quad = -2.0.(0/2) - 3 \times 0 = 0}$

$\mathbf{B.M\ at\ A\ point, x = 2,}$

$\mathbf{M_A = -2.x.(x/2) - 3 \times x}$

$\mathbf{M_A = -2 \times 2(2/2) - 3 \times 2 = -10\ KNm}$



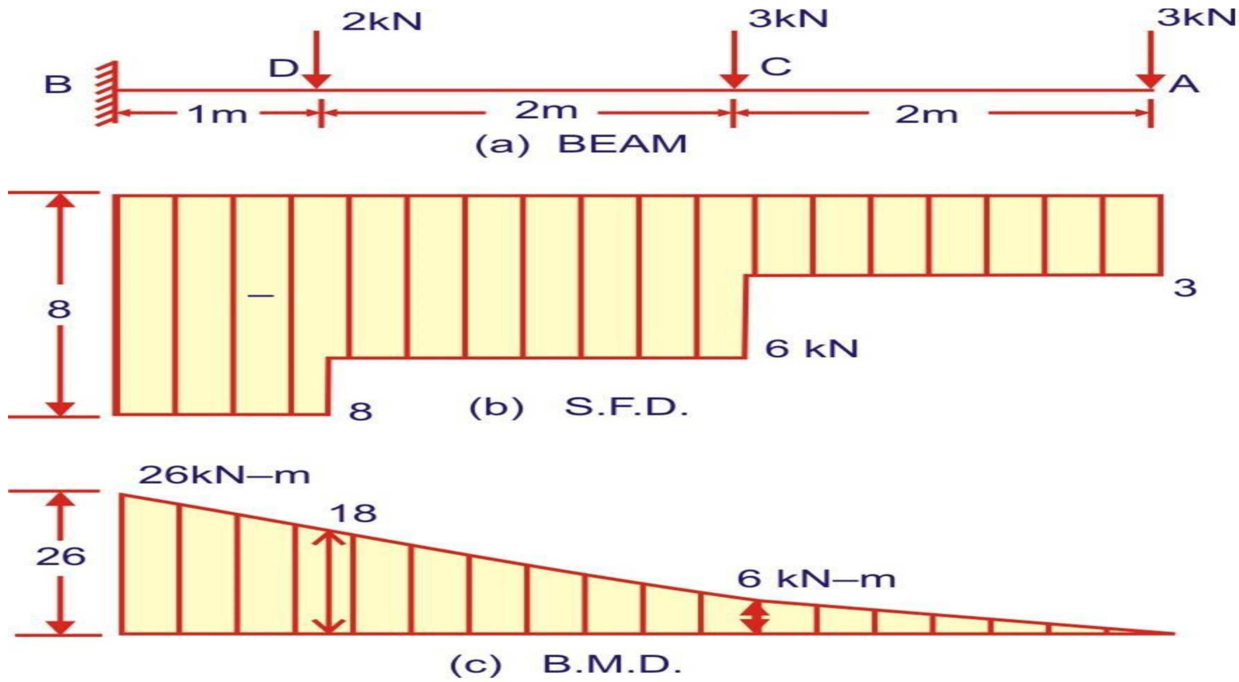


Fig. 3.9.

Shear Force Calculation:

Shear Force At Point B,

$S_A = -3\ KN$

Shear Force At Point D,

$S_C = -3 - 3 = -6\ KN$

Shear Force At Point C,

$S_D = -6 - 2 = -8\ KN$

Shear Force At Point A,

$S_B = -8\ KN$

Bending Moment Calculation:

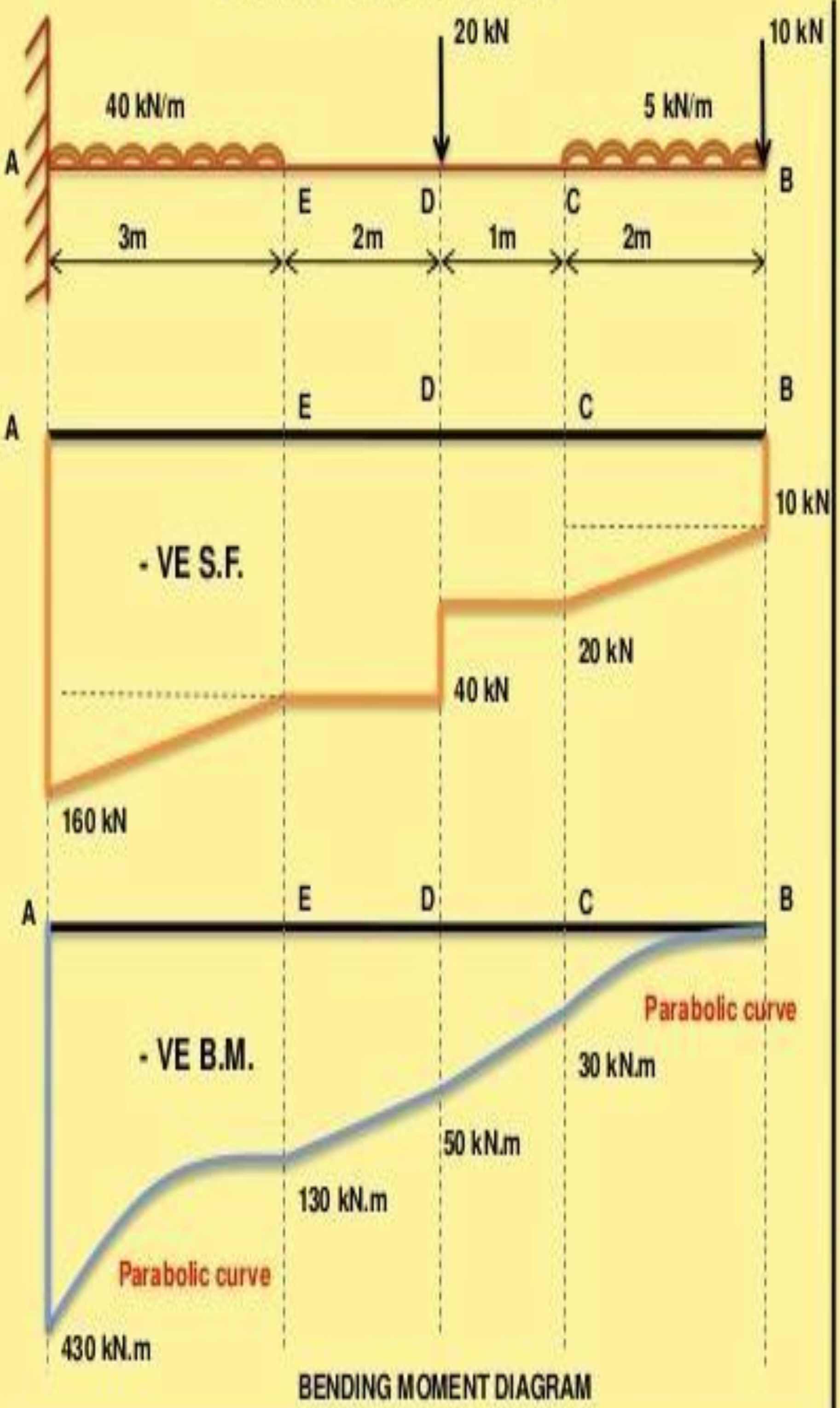
$\text{Bending Moment at Pont } A, M_A = +3 \times 0 = 0$

$\text{Bending Moment at Pont } C, M_C = +3 \times 2 = +6\ KNm$

$\text{Bending Moment at Pont } D, M_D = +3 \times 4 + 3 \times 2 = +18\ KNm$

$\text{Bending Moment at Pont } B, M_B = +3 \times 5 + 3 \times 3 + 2 \times 1 = 26KNm$

CANTILEVER WITH UDL



Calculations

Shear Force

S.F at B = 10 kN

S.F at C = 10 + 5 x 2 = -20 kN

S.F at D = 10 + 5 x 2 + 20 = - 40 kN

S.F at A = 10 + 5 x 2 + 20 + 40 x 3

= - 160 kN

Bending Moment

BM$_B$ = 0

BM$_C$ = 10 x 2 + 5 x 2 x 2/2 = -30 kNm.

BM$_D$ = 10 x 3 + 5 x 2 x (2/2 + 1)

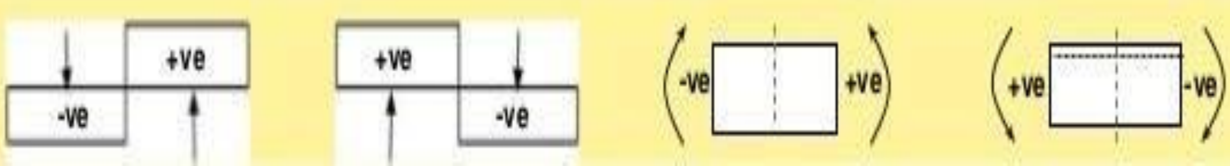
= 50 kN.m

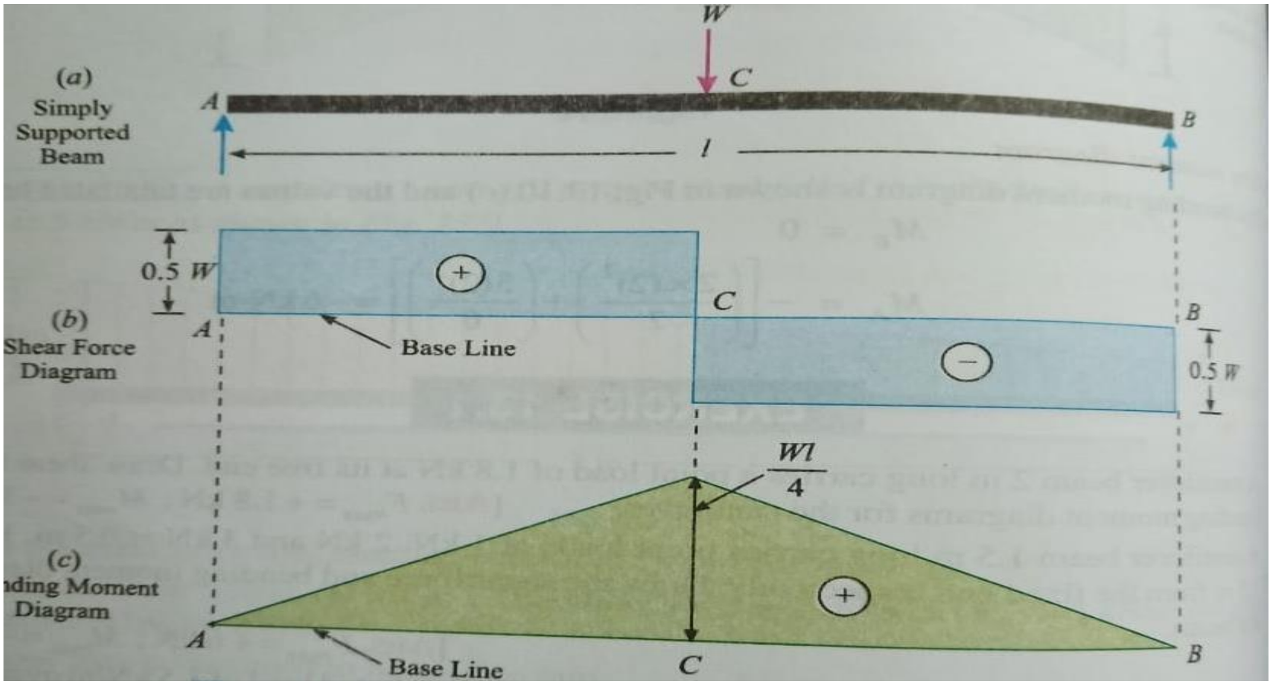
BM$_E$ = 10 x 5 + 5 x 2 x (2/2 + 3) + 20 x 2

= - 130 k Nm

BM$_A$ = 10 x 8 + 5 x 2 x (2/2 + 6) + 20 x 5 + 40 x 3 x 3/2

= - 430 kN.m.





Simple Supported Beam of length 'l' carrying Point Load W at its Centre:-

Consider a Simple Supported beam AB of Span l and carrying point load W at its Centre at point 'C' as shown in diagram

Calculation End Reaction: $Total\ Raction\ by\ Supports, R_A + R_B = W$

Taking Moment about Point B, $\sum M_B = 0$

$$R_A \times l - W \times {}^{l}/_{2} = 0$$

$$\Rightarrow R_A = {}^{W}/_{2}, \therefore R_B = {}^{W}/_{2}$$

$Consider\ a\ section\ X - X$

$at\ a\ distance\ of\ 'x'\ from\ the\ free\ end$

$S.F.at\ X - X\ section$

$$S_X = -{}^{W}/_{2},$$

$$S.F\ \ at\ \ Point\ B, S_B = -{}^{W}/_{2}$$

$$S.F\ \ at\ \ Point\ C, S_C = -{}^{W}/_{2} + W = +{}^{W}/_{2}$$

$$S.F\ \ at\ \ Point\ A, S_A = +{}^{W}/_{2}$$

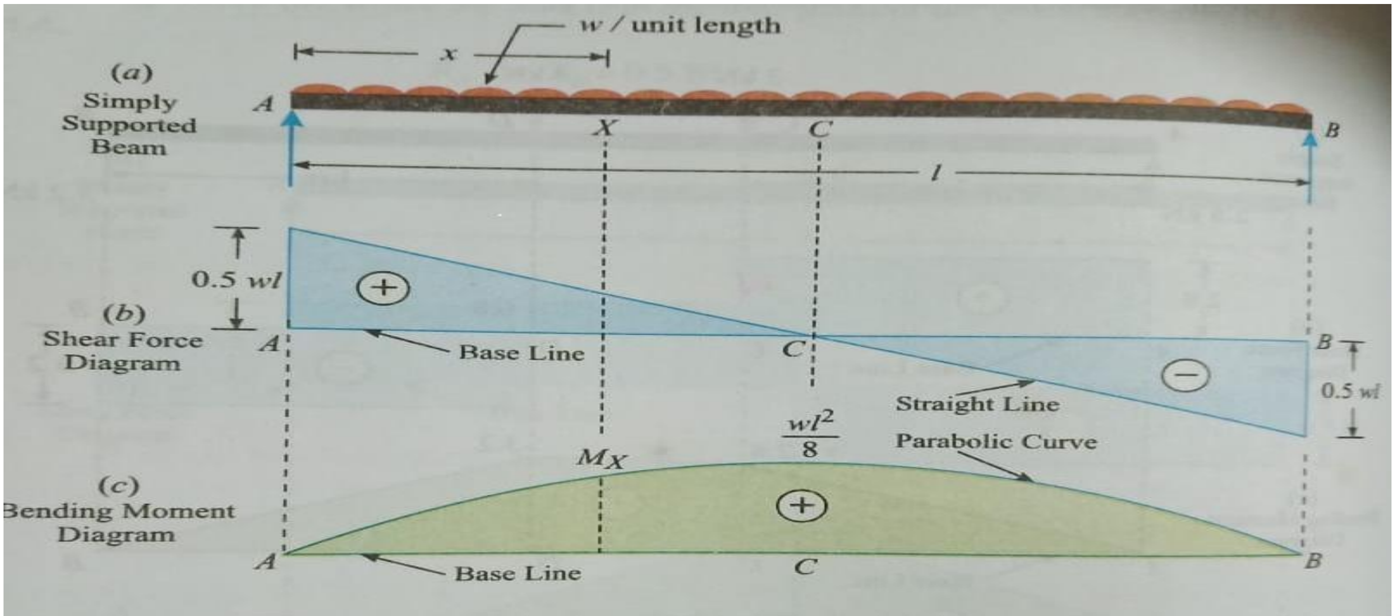
$Bending\ moment\ at\ X - X\ Section$

$$M_X = +{}^{W}/_{2}.x$$

$$B.M\ at\ B\ point, x = 0, M_B = +{}^{W}/_{2}.0 = 0$$

$$B.M\ at\ C\ point, x = {}^{l}/_{2}, M_C = +{}^{W}/_{2}.{}^{l}/_{2} = {}^{Wl}/_{4}.$$

$$B.M\ at\ A\ point, x = l, M_A = +{}^{W}/_{2}.l - {}^{W}/_{2}.l = 0$$



Simply supported beam with a uniformly distributed load

Simple Supported Beam of length 'l' carrying UDL W/m run:

Consider a Simple Supported beam AB of Span l and carrying UDL w/m run as shown in diagram

Calculation End Reaction: $Total\ Raction\ by\ Supports, R_A + R_B = Wl$

Taking Moment about Point B, $\sum M_B = 0$

$$R_A \times l - Wl \times {}^{l}/_{2} = 0$$

$$\Rightarrow R_A = {}^{Wl}/_{2}, \therefore R_B = {}^{Wl}/_{2}$$

$Consider\ a\ section\ X - X$

$at\ a\ distance\ of\ 'x'\ from\ the\ free\ end$

$S.F.\ at\ X - X\ section$

$S_X = -{}^{Wl}/_{2} + Wx,$

$S.F\ at\ Point\ B, S_B = -{}^{Wl}/_{2}$

$S.F\ at\ Point\ C, S_C = -{}^{Wl}/_{2} + {}^{Wl}/_{2} = 0$

$S.F\ at\ Point\ A, S_A = +{}^{Wl}/_{2}$

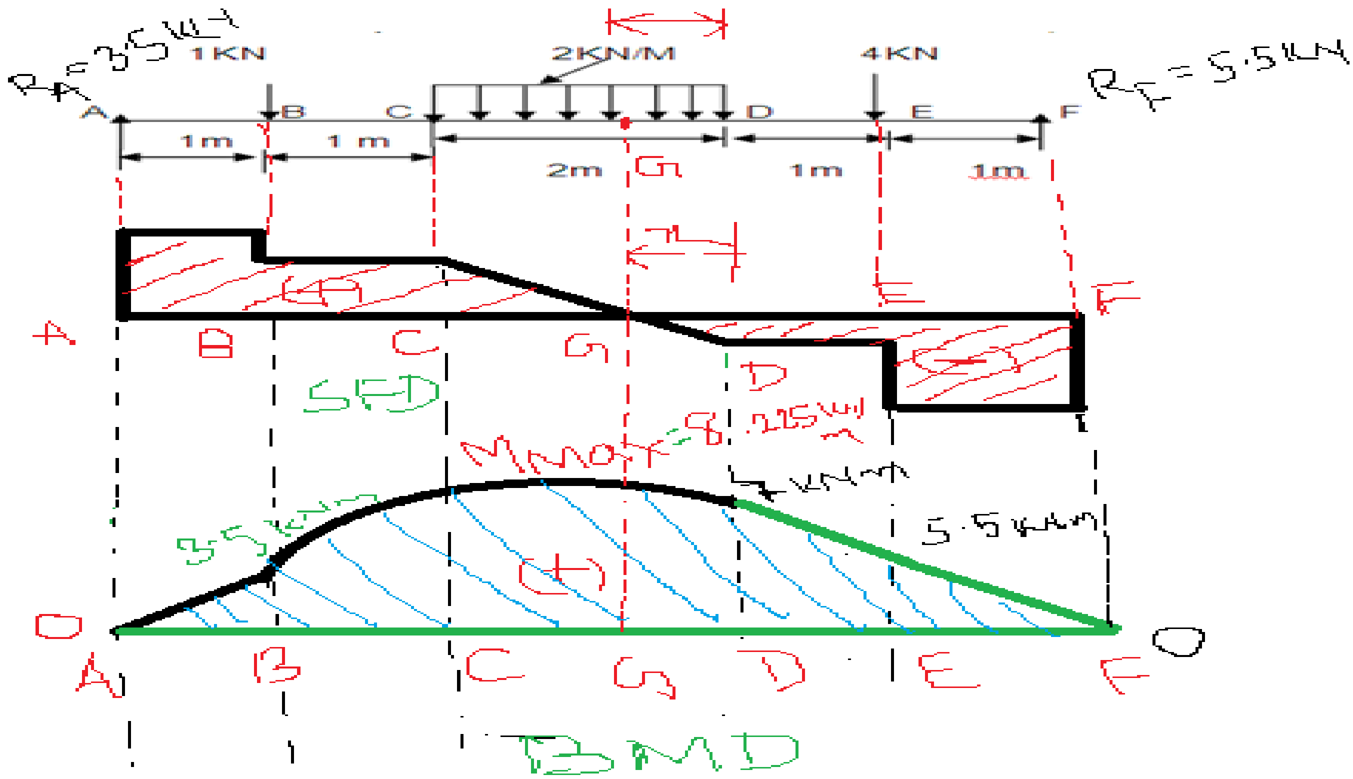
$Bending\ moment\ at\ X - X\ Section$

$M_X = +{}^{Wl}/_{2}.x - W\,{}^{x^2}/_{2}$

$B.M\ at\ B\ point, x = 0, M_B = +{}^{Wl}/_{2} \times 0 - W \times {}^{0^2}/_{2} = 0$

$B.M\ at\ C\ point, x = {}^{l}/_{2}, M_C = +{}^{Wl}/_{2} \times {}^{l}/_{2} - W \times {}^{({}^{l}/_{2})^2}/_{2} = +\frac{Wl^2}{8}$

$B.M\ at\ A\ point, x = l, M_A = +{}^{Wl}/_{2} \times l - W \times {}^{(l)^2}/_{2} = 0$



Calculation End Reaction:

$$Total\ Reaction\ by\ Supports, R_A + R_F = 9KN$$

Taking Moment about Point B, $\sum M_F = 0$

$$R_A \times 6 - 1 \times 5 - 2 \times 2 \times 3 - 4 \times 1 = 0$$

$$\Rightarrow R_A = {}^{5+4+12}/_{6} = 3.5\ KN$$

$$\therefore R_A = 3.5\ KN, \qquad R_F = 5.5\ KN$$

Shear Force Calculation

- ❑ $Shear\ Force\ At\ Point\ , S_F = -5.5KN$
- ❑ $Shear\ Force\ At\ Point\ E, S_E = -5.5 + 4 = -1.5\ KN$
- ❑ $Shear\ Force\ At\ Point\ E, S_E = -5.5 + 4 = -1.5\ KN$
- ❑ $Shear\ Force\ At\ Point\ C,\ S_C = -1.5 + 4 = +2.5\ KN$
- ❑ $Shear\ Force\ At\ Point\ B, S_B = +2.5 + 1 = +3.5\ KN$
- ❑ $Shear\ Force\ At\ Point\ A, S_A = +2.5 + 1 = +3.5\ KN$

Bending Moment Calculation:

- ❑ Bending Moment $At\ Point\ , M_F = 0$
- ❑ Bending Moment $At\ Point\ E, M_E = 5.5 \times 1 = -5.5\ KNm$
- ❑ Bending Moment $At\ Point\ D, M_D = 5.5 \times 2 - 4 \times 1 = 7\ KNm$
- ❑ Bending Moment $At\ Point\ C,\ M_C = 3.5 \times 2 - 1 \times 1 = 6KNm$
- ❑ Bending Moment $At\ Point\ B, M_B = +3.5 \times 1 = +3.5\ KNm$
- ❑ Bending Moment $At\ Point\ A, M_A = +3.5 \times 0 = 0$

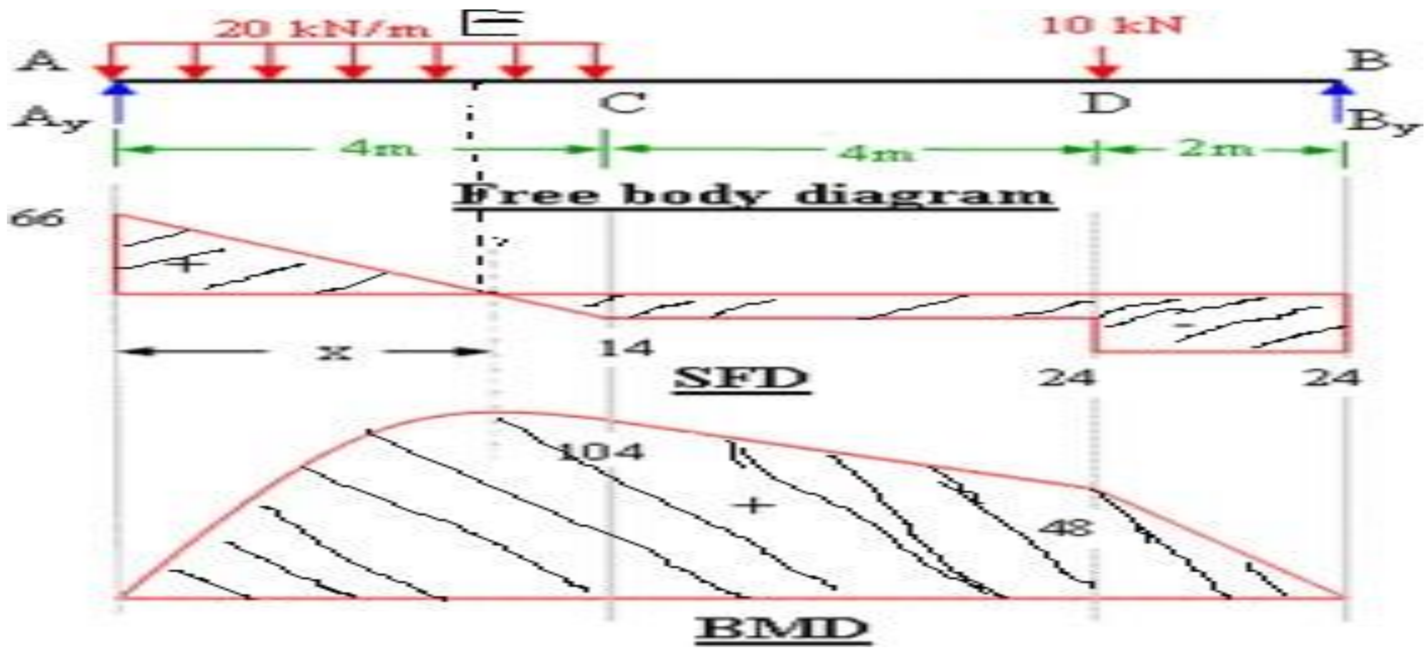
Maximum Bending Moment :

Maximum bending moment occurs at Point G , at a distance of 'x' from Point D

$$Shear\ Force\ At\ Point\ G, S_G = 2 \times x - 5.5 + 4 = 0 \Rightarrow x = 0.75m$$

$$Bending\ Moment\ at\ Pont\ G, M_G = 5.5 \times 2.75 - 4 \times 1.75 - 2 \times 0.75 \times \left({}^{0.75}/_{2}\right) = 8.225KNm$$

$$M_{Max} = 8.225KNm$$



Calculation End Reaction:

$$Total\ Reaction\ by\ Supports, R_A + R_B = 90KN$$

Taking Moment about Point B, $\sum M_B = 0$

$$R_A \times 10 - 10 \times 2 - 20 \times 4 \times 8 = 0$$

$$\Rightarrow R_A = {10 \times 2 + 20 \times 4 \times 8}/_{10} = 66\ KN$$

$$\therefore R_A = 66KN, \qquad R_B = 24\ KN$$

Shear Force Calculation

$Shear\ Force\ At\ Point\ B, S_B = -24\ KN$

$Shear\ Force\ At\ Point\ D, S_D = -24 + 10 = -14\ KN$

$Shear\ Force\ At\ Point\ C,\ S_C = -24 + 10 = -14\ KN$

$Shear\ Force\ At\ Point\ A, S_A = -14 + (20 \times 4) = +66\ KN$

Bending Moment Calculation:

$$Bending\ Moment\ at\ Pont\ B, M_B = +24 \times 0 = 0$$

$$Bending\ Moment\ at\ Pont\ D, M_D = +24 \times 2 - 10 \times 0 = +48\ KNm$$

$$Bending\ Moment\ at\ Pont\ C, M_C = +24 \times 6 - 10 \times 4 = +104\ KNm$$

$$Bending\ Moment\ at\ Pont\ A, M_A = +24 \times 10 - 10 \times 8 - (20 \times 4 \times 2) = 0$$

Maximum Bending Moment :

Maximum bending moment occurs at Point E, at a distance of 'x' from Support - A $\sum M_A = 0$

$$Shear\ Force\ At\ Point\ E, S_E = 20 \times (4 - x) + 10 - 24 = 0 \Rightarrow x = 3.3m$$

$$Maximum\ Bending\ Moment\ at\ Pont\ E, M_E = 66 \times 3.3 - 20 \times 3.3 \times \left({3.3}/_{2}\right) = 108.90KNm$$

$$M_{Max} = 108.90KNm$$

Concept of Maximum Bending Moment

1. It occurs at a point in Shear Force diagram where the value Shear Force is Zero in a Simple Supported Beam
2. The projection of point on the Bending Moment Diagram will give the value of maximum value of Bending moment acting on the beam

- ❑ For cantilever Beam Subjected to point Load at its Free end

$$M_{max} = WL$$

- ❑ cantilever Beam Subjected to Uniformly Distributed Load over whole Span

$$M_{max} = {WL^2}/{2}$$

- ❑ Simple Supported Beam Subjected to Point Load at its Center of length of Span

$$M_{max} = {WL}/{4}$$

- ❑ Simple Supported Beam Subjected to Point Load at its Center of length of Span

$$M_{max} = {WL^2}/{8}$$

Chapter-5
Theory of Simple Bending

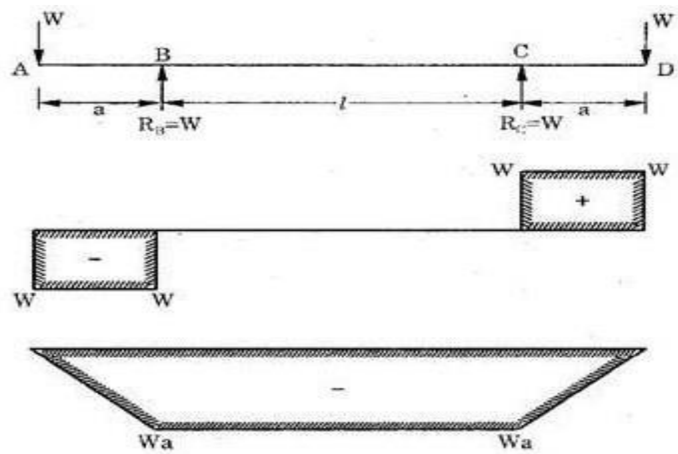
Introduction:

In previous chapter of Shear Force and Bending moment when the beam is loaded with some vertical external forces it deflects or tends to bend and calculate the Shear force and Bending Moment at each section of beam representing them graphically

Here in this chapter we will go for study finding out the Internal Stresses developed in beam while they are subjected to different loads (Point Load and Uniformly distributed load)

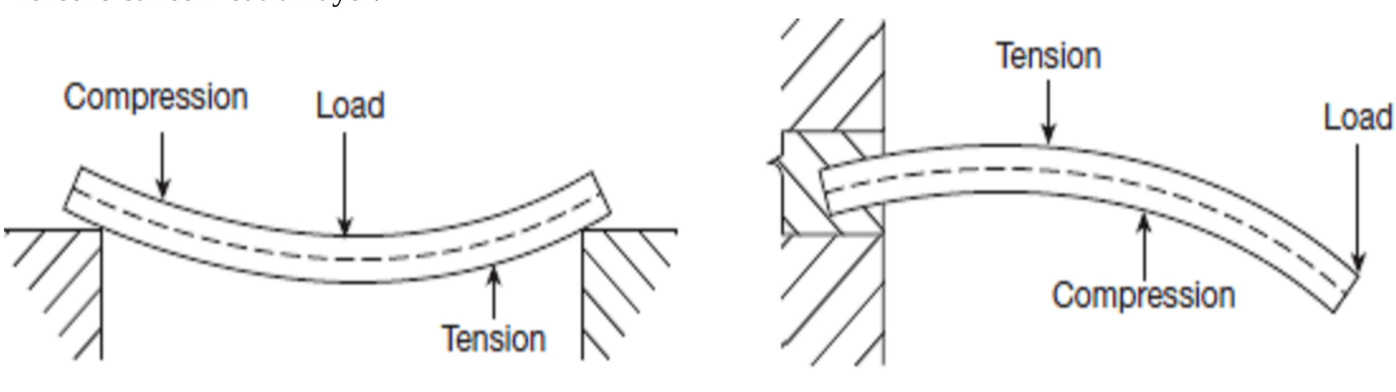
Definition of Pure Bending:

In case of overhanging beam whenever loads are applied at its overhang portions Then it is observed that from (SF & BM Diagram) the sections having same shear force value at the two ends of supports, but there is a portion found between two supports where the value of shear force is zero and bending moment remains constant, this portion is subjected to pure bending.



❖ Theory of Simple Bending:

- When a beam is subjected to two equal and opposite couples at its two ends, then it causes the beam to bend in to Concave and Convex shape before its failure taking place.
- It is observed that the layers of beam expands or contracts in length from its longitudinal axis.
- Compressive force is exhibited on the layer having smaller radius of curvature & tensile force is exhibited on the layer having larger radius of curvature.
- But it is found that at the longitudinal axis of beam layer are not subjected to neither tensile nor compressive force is called Neutral layer.



(a) Simply supported beam (b) Cantilever beam

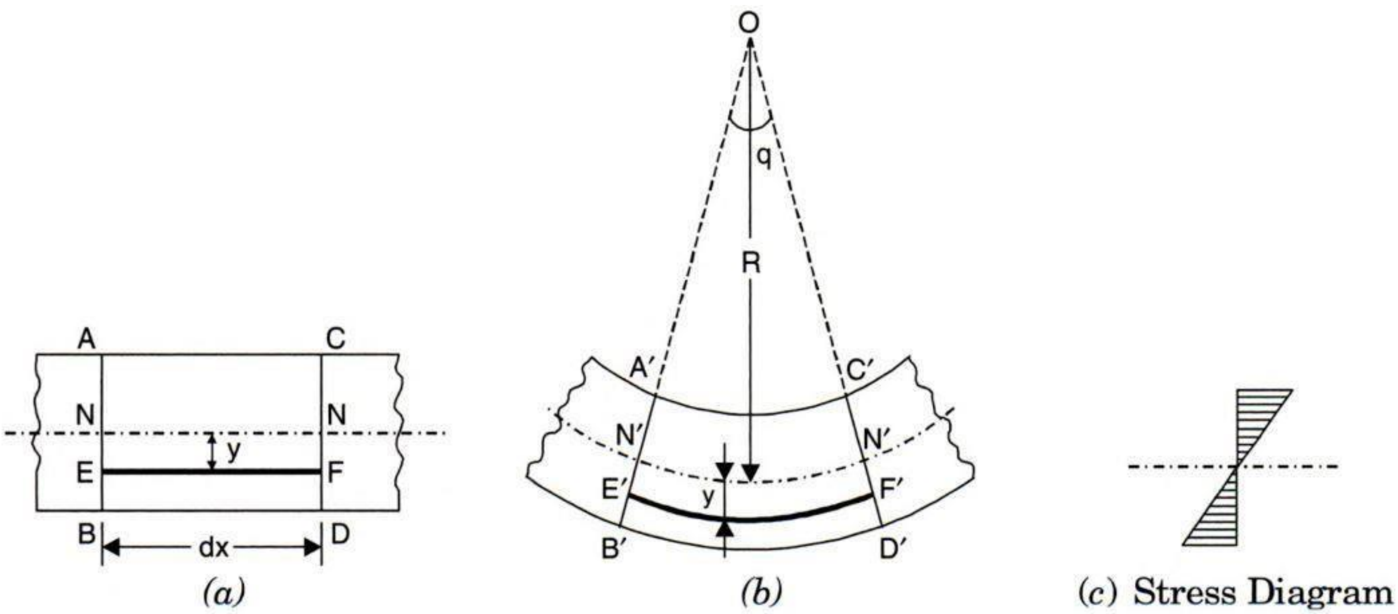
Assumption taken in Simple Bending:

i. The material of beam is perfectly homogeneous throughout.
ii. The stress induced is proportional to the strain and at no place the stress exceeds the elastic limit.
iii. The value of modulus of elasticity is same fibres of the beam under compression or tension.
iv. The transverse section of the beam, which is plane before bending, remains plane after bending.

v. The loads are applied in the plane of bending.

vi. The radius of curvature of the beam before bending is very large in comparison to it transverse dimension

Derive of Bending Equation.



Let's consider a beam of rectangular shape 'ABCD' as shown in above diagram is subjected to two equal and opposite couples at its two ends causing the beam to bend into concave shape as fig. b consider a layer EF at a distance of 'y' form longitudinal axis 'NN'

The layers AC changes its length to A'C' and layer BD changes to B'D'

The layer EF is also changing its length to E'F' as shown in fig (b)

Let

$\theta - angle\ of\ arc$

$R - Radius\ of\ Curvature\ of\ Beam$

$E - Young's\ Modulus\ of\ Elasticity\ for\ material\ of\ Beam$

Original length of layers before Bending

$$AC = BD = NN = EF = R\theta$$

Since the layer EF is on the tensile zone of Bending Beam and there by its length increases

$$Change\ in\ length\ of\ layer\ EF, \delta l = E'F' - EF = (R+Y)\theta - R\theta = R\theta$$

$$Strain\ in\ layer\ EF = \frac{\delta l}{l} = \frac{E'F' - EF}{EF} = \frac{Y\theta}{R\theta} = \frac{Y}{R}$$

From Hook's law of Elasticity stress is directly proportional to Strain

i.e $\sigma \propto e \Rightarrow \sigma = E.e$

Now bending stress

$\Rightarrow \sigma_b = E.\frac{Y}{R} \ldots\ldots\ldots\ldots\ldots\ldots (i)$ $(putting\ value\ of\ \ strain\ e = \frac{Y}{R})$

$$\Rightarrow \frac{\sigma_b}{Y} = \frac{E}{R} \ldots\ldots\ldots\ldots\ldots\ldots \quad (i)$$

Moment of Resistance (M):

It is defined as the resistance offered by the material of beam to that of the applied moment by the vertical loads acting along the longitudinal direction of beam or it is the value of bending moment produced by the shear forces acting on beam

For finding out the total moment of resistance offered by the beam, let's consider the cross-section of beam convert it in to no of equal division of Area 'dA' and thickness of strip 'dY' let the strip is at a distance of 'Y' from neutral axis 'NN'

Bending Stress acting on Small strip

$$\Rightarrow \sigma_b = E.\frac{Y}{R}$$

$$Force\ acting\ on\ Small\ Strip, dF = \sigma_b.dA$$

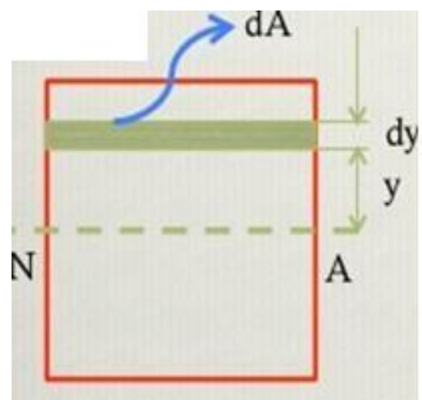
$$Moment\ of\ Force\ acting\ on\ Small\ Strip\ about\ Neutral\ Axis, dM = dF.Y = \left(E.\frac{Y}{R}.dA\right)Y$$

$$Total\ Moment\ produced\ by\ the\ Force\ on\ beam\ about\ neutral\ axis\ M = \int dM$$

$$M = \int \left(E.\frac{Y}{R}.dA\right)Y = \frac{E}{R}\int Y^2 dA = \frac{E}{R}I$$

$$\frac{M}{I} = \frac{E}{R} \ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

Now Equating Both Equation (i) and (ii)



$$\Rightarrow \frac{\sigma_b}{Y} = \frac{E}{R} \ldots\ldots\ldots\ldots\ldots\ldots \quad (i)$$

$$\frac{M}{I} = \frac{E}{R} \ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

$$\boldsymbol{\frac{M}{I} = \frac{E}{R} = \frac{\sigma_b}{Y} \ldots\ldots\ldots\ldots\ldots Bending\ Equation}$$

Where

$M -$ *Moment of Resistance offered by Beam*

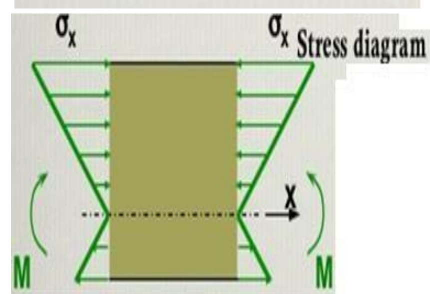
$I -$ *Moment of inertia of Beam About Neutral Axis*

$E -$ *young's Modulus for Material of Beam*

$R -$ *Radius of Curvature*

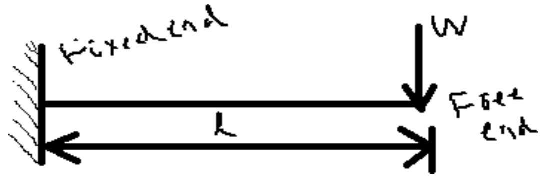
$\sigma_b -$ *Bending Stress induced in beam*

$Y -$ *Distance of outermost layer from Neutral Axis* $N - N$



Moment of Resistance for Beam subject to load

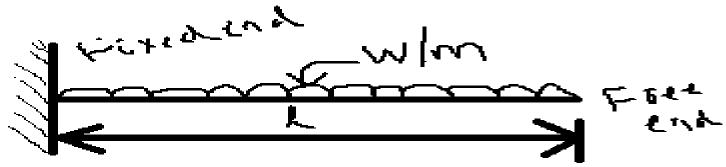
- For cantilever Beam Subjected to point Load at its Free end



$$M_{max} = WL$$

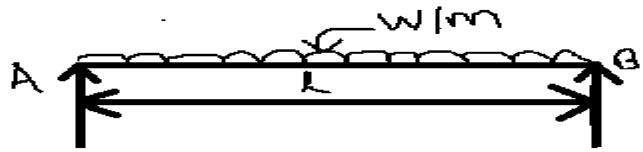
- cantilever Beam Subjected to Uniformly Distributed Load over whole Span

$$M_{max} = {WL^2}/{2}$$



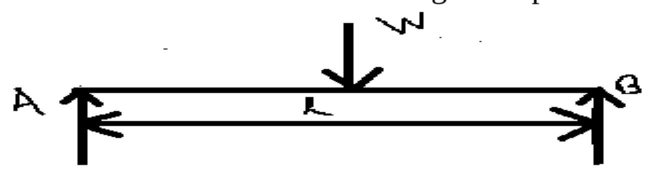
- Simple Supported Beam Subjected to Point Load at its Center of length of Span

$$M_{max} = {WL}/{4}$$



- Simple Supported Beam Subjected to Uniformly Distributed Load at its Center of length of Span

$$M_{max} = {WL^2}/{8}$$



Sectional Modulus:

It is defined as the ratio of moment of inertia of body to the maximum distance of outer fibre from central axis of beam or column.

It is denoted by symbol ‘Z’

$$Z = \frac{I}{y}$$

$Z - section\ modulus\ of\ column$

$I - M.O.I\ of\ column\ along\ vertical\ direction$

y- Distance of outer fibre from central axis

Its Unit is mm^3

For rectangular section $Z = \frac{b*d^2}{6}$

For circular section $Z = \frac{\pi}{32} * d^3$

By using section modulus the strengt5h of beam is calculated

The bending stress induced in beam

$\sigma_b = \frac{M}{Z}$

Where M- bending moment due to load on beam, $\sigma_b -\ bending\ stress\ in\ duced\ in\ beam$

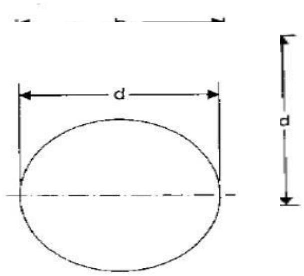
1. Rectangular Section

3. Circular Section

$$I = \frac{\pi}{64}\, d^4$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{\pi}{32}\, d^3$$

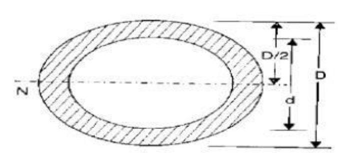


4. Circular Hollow Section

$$I = \frac{\pi}{64}\ [D^4 - d^4]$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{\pi}{32D}\ [D^4 - d^4]$$



Bending Stress:

It is defined as the ratio of Moment of Resistance Offered by beam to the section modulus of beam.
It is denoted by symbol 'σ_b'

Mathematically

Bending Stress

$$\sigma_b = \frac{moment\ of\ resistance\ offered\ by\ beam}{Section\ Modulus\ of\ Beam} = \frac{M}{Z}$$

$$\sigma_b = \frac{M}{Z}$$

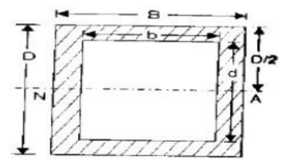
It is expressed in unit $N/_{mm^2}$

2. Rectangular Hollow Section

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$y_{max} = \left(\frac{D}{2}\right)$$

$$Z = \frac{1}{6D}\left[BD^3 - bd^3\right]$$



Q. A rectangular beam 8cm x 6 cm is 2 m long and is simply supported at the ends. If carries a load of 3kN at mid-span. Determine the maximum bending stress induced in the beam. [2014wn]

Given data:

$$Type\ of\ Beam - Simple\ Supported$$
$$Type\ of\ Load - Point\ load$$
$$d = 80\ mm$$
$$b = 6\ cm = 60\ mm$$
$$l = 2m = 2000\ mm$$
$$W = 3KN = 3000\ N$$
$$Maximum\ bending\ Stress = ?$$

We have for a Simple Supported Beam carrying Point Load at Centre

$$M = \frac{Wl}{4} = \frac{3000 \times 2000}{4} = 15 \times 10^5 Nmm$$

Section Modulus for Rectangle Cross-section

$$Z = \frac{b \times d^2}{6} = \frac{60 \times 80^2}{6} = 64000\ mm^3$$

Now bending stress induced by beam

$$\sigma_b = \frac{M}{Z} = \frac{15 \times 10^5}{64000} = 23.43\ N/_{mm^2}$$

Q. **A rectangular beam 300mm deep is simply supported over a span of 4m what uniformly distributed load the beam may carry if the bending stress is not to exceed 120MPa. Take I=225x10^6mm^4 [2015w]**

Given data:

$$Type\ of\ Beam - Simple\ Supported$$

$Type\ of\ Load - Uniformly\ Distributed\ load$
$d = 300\ mm$
$l = 4m = 4000\ mm$
$I = 225 \times 10^6 mm^4$
$W = ?$
$Maximum\ bending\ Stress = 120\ MPa = 120\ {}^{N}/_{mm^2}$

Section Modulus for Rectangle Cross-section

$$Z = \frac{I}{y} = \frac{225 \times 10^6}{300/2} = 15 \times 10^6 mm^3$$

Now bending stress induced by beam

$$\sigma_b = \frac{M}{Z}$$

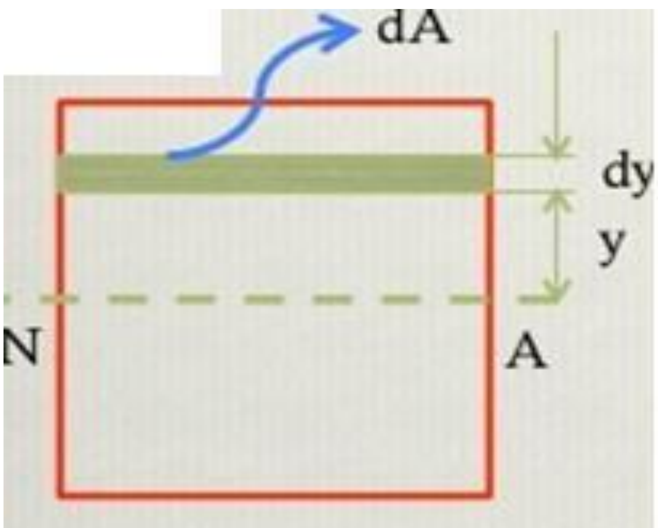
$$\Rightarrow M = \sigma_b \times Z = 120 \times 15 \times 10^6 = 180 \times 10^7 Nmm$$

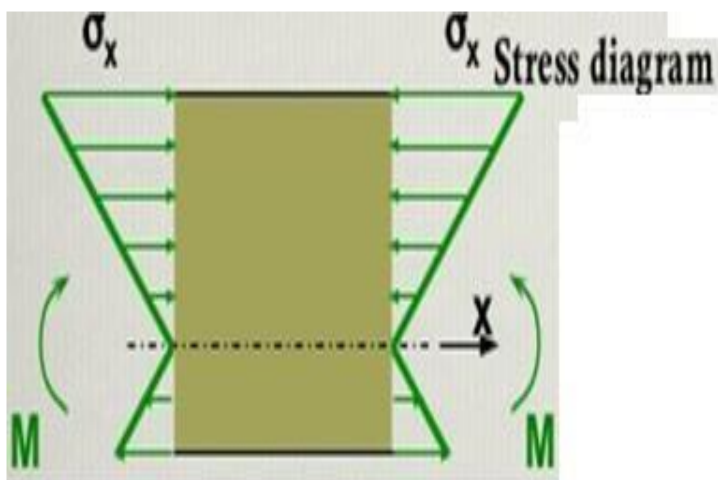
We have for a Simple Supported Beam carrying UDL

$$M = \frac{Wl^2}{8} = 180 \times 10^7$$

$$\Rightarrow W = \frac{180 \times 10^7 \times 8}{4000^2} = 90 \frac{N}{mm}$$

- **Position of neutral axis is passing through central axis:**





Let’s consider the cross-section of beam is subject to bending moment as shown in diagram
In order to prove the neutral axis coincides with central axis of beam
Let’s consider the equilibrium of cross-section of beam
$dA - small\ strip\ of area$

y- Distance of small strip from centre axis
Now force acting on small strip

$df = \sigma_b . dA$

$\Rightarrow df = \frac{E}{R} . y . dA$ ($\because$ $from\ bending\ moment\ equaion\ \frac{M}{I} = \frac{E}{R} = \frac{\sigma_b}{y}$)

Now force acting on total cross-section

F= $\int df = \int \frac{E}{R} . y . dA = \frac{E}{R} \sum y . dA$

For equilibrium of section $\sum y . dA = 0$

Now F=0

As the value of F=0, moment is also zero now it is proved that the neutral axis is passing through the centre of area & neutral axis coincides with the centre axis of beam

Flexural Rigidity:

It is defined as the bending moment required to produce unit radius of curvature

We know that from bending moment equation

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma_b}{y}$$

$$\Rightarrow \frac{M}{I} = \frac{E}{R}$$

$$\Rightarrow M = \frac{E.I}{R}$$

Where R- radius of curvature, M- bending moment

Where the product of young's modulus of elasticity and moment of inertia of beam section is called flexural rigidity ($E.I$)

Its unit Nm^2

Q. A rectangular beam 200mm deep is simply supported over a beam of span 2m. Find the uniformly distributed load, the beam can carry if the bending stress is not to exceed 25MPa. Take / For the beam as 8 x 10^6 mm^4. [2017, 2014, 2015]

Given data:

$Type\ of\ Beam - Simple\ Supported$

$Type\ of\ Load - Uniformly\ Distributed\ load$

$d = 200\ mm$

$l = 2m = 2000\ mm$

$I = 8 \times 10^6 mm^4$

$W = ?$

$Maximum\ bending\ Stress = 25\ MPa = 25\ {}^{N}/_{mm^2}$

Section Modulus for Rectangle Cross-section

$$Z = \frac{I}{y} = \frac{8 \times 10^6}{200/2} = 8 \times 10^4 mm^3$$

Now bending stress induced by beam

$$\sigma_b = \frac{M}{Z}$$

$$\Rightarrow M = \sigma_b \times Z = 25 \times 8 \times 10^4 = 200 \times 10^4 Nmm$$

We have for a Simple Supported Beam carrying UDL

$$M = \frac{Wl^2}{8} = 200 \times 10^4$$

$$\Rightarrow W = \frac{200 \times 10^4 \times 8}{2000^2} = 4\frac{N}{mm} = 4000\frac{N}{m}$$

Q. A simply supported beam of length 8m is loaded with uniformly distributed load of 3kN/m over the entire span. If the bending stress of the beam is not to exceed 20N/mm^2, find the dimension of the beam. Assume that the width of the beam is half of the depth of the beam [2016]

Ans.:

Given data:

$$Type\ of\ Beam - Simple\ Supported$$
$$Type\ of\ Load - Uniformly\ Distributed\ load$$
$$d = ?, b = \frac{d}{2}$$
$$l = 8m = 8000\ mm$$
$$E = 200\ GPa$$
$$W = 3\ KN/m$$
$$Maximum\ bending\ Stress = 2\ {}^{N}/_{mm^2}$$

We have for a Simple Supported Beam carrying UDL

$$M = \frac{Wl^2}{8} = \frac{3 \times 1000 \times 8000^2}{8} = 24 \times 10^9 Nmm$$

Now Section modulus of beam

$$\sigma_b = \frac{M}{Z}$$

$$\Rightarrow Z = \frac{M}{\sigma_b} = \frac{24 \times 10^9}{2} = 12 \times 10^9 mm^3$$

Considering Moment of Resistance:

Section Modulus for Rectangle Cross-section

$$\Rightarrow Z = \frac{bd^2}{6} = 12 \times 10^9 mm^3$$

$$\Rightarrow Z = \frac{\frac{d}{2} \times d^2}{6} = 12 \times 10^9 mm^3$$

$$\Rightarrow \frac{\boldsymbol{d^3}}{\boldsymbol{12}} = 12 \times 10^9$$

$$\Rightarrow d = \sqrt[3]{12 \times 10^9 \times 12} = 5241.48\ mm\ Say\ 5.241m \ldots\ldots\ldots\ldots\ldots Depth\ of\ Beam$$

$$Now\ width\ of\ beam\ b = {}^{d}/_{2} = {}^{5241.48}/_{2} = 2620.74mm\ say\ 2.620m$$

$\boldsymbol{Dimensions\ of\ Beam\ are\ d = 5.241\ m}$ & $\boldsymbol{b = 2.620\ m}$

Chapter-6

Combined Direct & Bending Stress

INTRODUCTION:

Often we come across members carrying longitudinal thrust or pull as in the case of a pillar or a tie-rod. Sometimes, these members are also subjected to bending stresses in addition to the direct stresses. In this chapter, we shall study the effect of these stresses on the cross-section of the members and also, the stress-distribution across the cross-section. We conclude with the application as this theory to real situations such as walls and pillars subjected to wind forces.

Objectives after studying this unit,

- Students should be able to calculate the stresses for different eccentrically loaded cross section,
- Explain the Middle Third Rule for no tension condition,
- Analyses the effect of wind forces and their stress distribution pattern on masonry walls, pillars and tall chimneys, and design sections for members carrying direct compressive force and bending stresses.

- **Column:**

  Definition:

  - A bar or a member of a structure inclined at 90° to the horizontal and carrying an axial compressive load is called a column.
  - It is also defined as the structural member which always held vertically to support the beams resting upon it.
  - It is primarily subjected to axial Compressive loads.

Axial Load:

It is the load whose line of action coincides with the longitudinal axis of Column. Which is always compressive in nature while acting on Column.

It is denoted by symbol ‘P’, Unit- KN

Axial Stress/Direct Stress:

It is defined as the ratio of axial load per unit area of Cross-Section of Column. As shown in fig a.

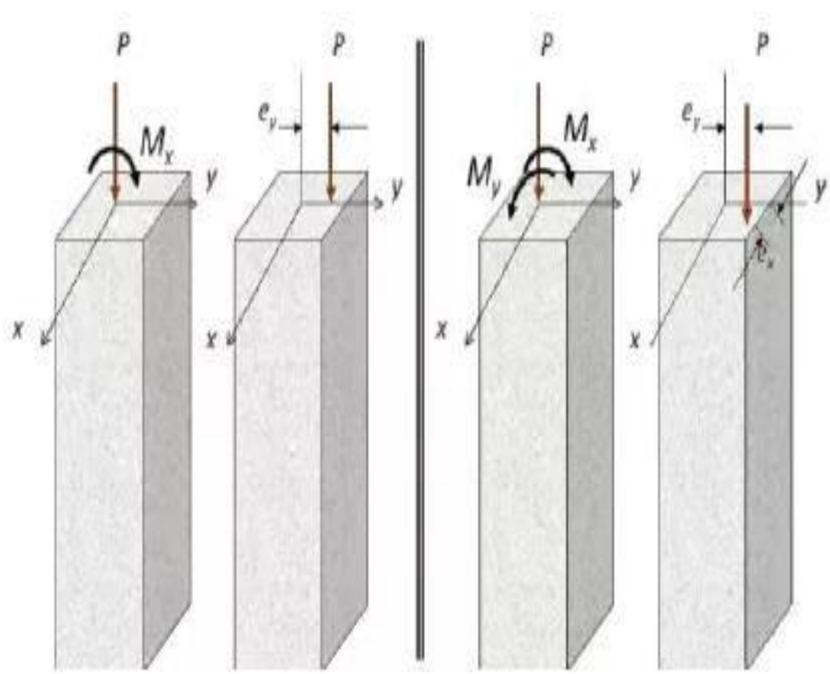
It is denoted by Symbol ‘σ_o’

Unit: $N/_{m^2}$, KPa

Eccentric Load:

It is defined as the type of load whose line of action is at an eccentricity with the longitudinal vertical axis of column. As shown in fig b. & c.

Due to this load the column may tend to bend



It is denoted by symbol ‘P’, Unit- KN

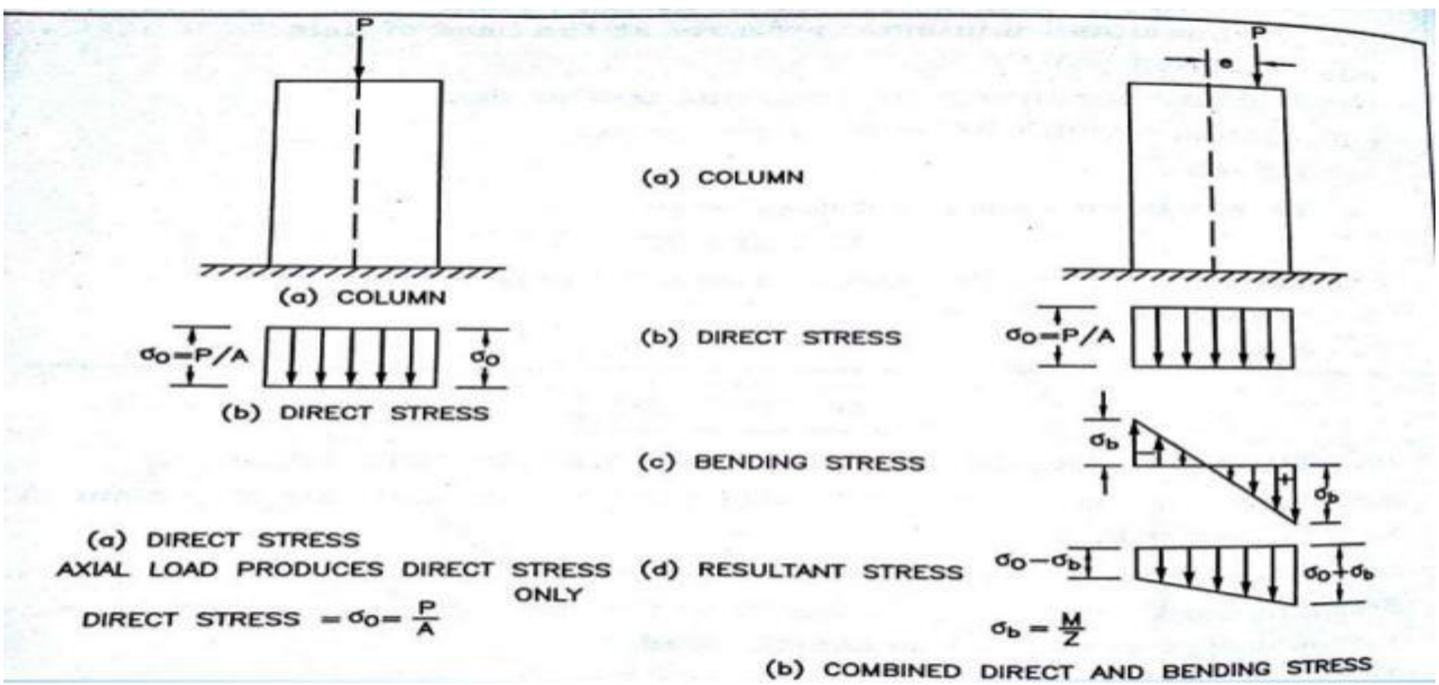
Effect of Eccentric Load on Column:

Let’s consider a column is loaded eccentrically as shown in diagram. The load ‘P’ is applied with an eccentricity ‘e’ from vertical axis of column. The load is acting on the plane which bisects thickness of column.

Due to this eccentric load the column may deflected along the right side more and causes the bending from its fixed position at Foundation. The column dips more in right side and tries to leave from foundation with its left part.

The column should have safely carrying this load without its fracture. In this way the couple forces are induced at the both side of column from the fixed position at foundation which leads to develop the bending stress in column

The right part of column is subjected to maximum effect of bending stress and compressive stress is induced. While on other hand the tensile stress leads to produce at the left side and is subjected minimum effect of Bending Stres



Due to eccentric load, the combined effect of both Axial and bending stress will be considered

We have due to axial load

1. The Axial stress developed in column given by

$$\sigma_o = \frac{P}{A}$$

Where P- load applied along Axis of Column

A- Area of Cross-section of Column

2. Due to Eccentric Load on Column

Bending Moment Produced by the Load

$M = P \times e$

$we\ have\ know\ that\ the\ Bending\ Stress\ induced\ in\ Column$

$$\sigma_b = \frac{M}{Z}$$

$where\ Z - Section\ Modulus\ of\ Cross - Section\ of\ Column\ about\ it's\ Longitudinal\ axis$

$$Z = \frac{Moment\ of\ Inertia\ of\ Cross\ Section\ of\ Column\ about\ Vertical\ Axis\ (Y - Y\ Axis)}{Distance\ of\ Outermost\ layer\ of\ Column\ form\ the\ Neutral\ Axis\ (Y - Y\ Axis)}$$

$$Z = \frac{I_{Y-Y}}{Y_{Max}} = \frac{\frac{d \times b^3}{12}}{b/2} = \frac{d \times b^2}{6}$$

$$\sigma_b = \frac{M}{Z} = P \times e \Big/ \frac{d \times b^2}{6} = \frac{6 \times P \times e}{d \times b^2}$$

$$\sigma_b = \frac{6 \times P \times e}{d \times b^2}$$

Now considering the combined effect of Both Direct Stress & Bending Stress on column

$Now\ Maximum\ Stress\ induced\ in\ column\ ,\qquad \sigma_{Max} = \sigma_O + \sigma_b$

$$\sigma_{Max} = \frac{P}{A} + \frac{M}{Z}$$

$$\sigma_{Max} = \frac{P}{A} + \frac{6 \times P \times e}{d \times b^2} = \frac{P}{A} + \frac{6 \times P \times e}{d \times b \times b}$$

$$\sigma_{Max} = \frac{P}{A} + \frac{6 \times P \times e}{A \times b} = \frac{P}{A}(1 + \frac{6e}{b})$$

$$\boldsymbol{\sigma_{Max} = \frac{P}{A}(1 + \frac{6e}{b})} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \boldsymbol{Maximum\ Stress\ Induced\ in\ Column\ (Compressive)}$$

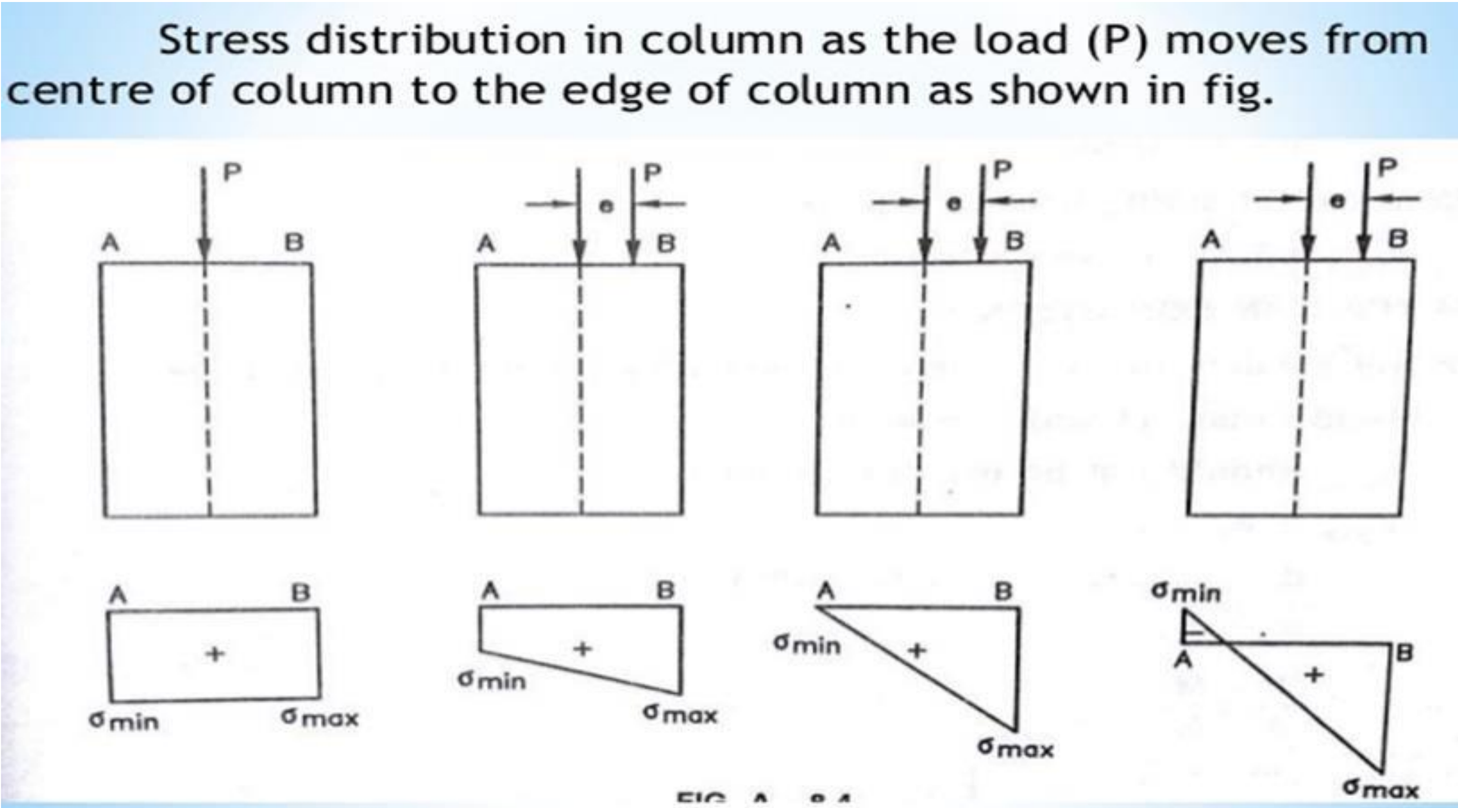
$Now\ Minimum\ Stress\ induced\ in\ column\ ,\qquad \sigma_{Min} = \sigma_O - \sigma_b$

$$\sigma_{Min} = \frac{P}{A} - \frac{M}{Z}$$

$$\sigma_{Min} = \frac{P}{A} - \frac{6 \times P \times e}{d \times b^2} = \frac{P}{A} - \frac{6 \times P \times e}{d \times b \times b}$$

$$\sigma_{Min} = \frac{P}{A} - \frac{6 \times P \times e}{A \times b} = \frac{P}{A}(1 - \frac{6e}{b})$$

$$\boldsymbol{\sigma_{Min} = \frac{P}{A}(1 - \frac{6e}{b})} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \boldsymbol{Minimum\ Stress\ Induced\ in\ Column\ (Tensile\ )}$$



1. **A circular cast iron column of diameter 250mm carries a vertical load of 600KN a distance of 35mm from the axis. Find the extreme values of the stresses induced in the section.**

Given data:

d =250 mm

P=600 KN =600000 N

e = 35 mm

For circular section of column

Area of Cross section of Column

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 250^2 = 49087.38\, mm^2$$

Section modulus of column

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32} 250^3 = 1533980.78\, mm^3$$

Direct compressive stress(σ_c):

We know that

$$\sigma_c = \frac{P}{A} = \frac{P}{\frac{\pi}{4} \times d^2} = \frac{6 \times 10^5}{\frac{\pi}{4} \times 250^2} = 12.22\, \frac{N}{mm^2}$$

Bending stress induced in column (σ_b):

We know that

$$\sigma_b = \frac{M}{Z} = \frac{P \times e}{\frac{\pi}{32} d^3} = \frac{600000 \times 35}{\frac{\pi}{32} 250^3} = 13.68\, \frac{N}{mm^2}$$

Maximum stress induced in beam due to eccentric load (σ_{max}):

$$\sigma_{max} = \sigma_c + \sigma_b = \frac{P}{A} + \frac{M}{Z} = \frac{P}{\frac{\pi}{4} \times d^2} + \frac{P \times e}{\frac{\pi}{32} d^3} = 12.22 + 13.68 \frac{N}{mm^2} = 25.88 \frac{N}{mm^2}$$

$$\boldsymbol{\sigma_{max} = 25.88 \frac{N}{mm^2} (Compressive)}$$

Minimum stress induced in beam due to eccentric load (σ_{min}):

$$\sigma_{min} = \sigma_c - \sigma_b = \frac{P}{A} - \frac{M}{Z} = \frac{P}{\frac{\pi}{4} \times d^2} - \frac{P \times e}{\frac{\pi}{32} d^3}$$

$$= 12.22 - 13.68 \frac{N}{mm^2} = -1.48 \frac{N}{mm^2} (Tensile)$$

$$\boldsymbol{\sigma_{min} = -1.48 \frac{N}{mm^2} (Tensile)}$$

4.* **A short column of dia. 2 meters is subjected to a compressive load of 2 tones at an eccentricity of 5 cm from its axis. Find the maximum stress developed. Sketch the distribution of stresses across the section.**

[2014wbp]

Given data:

d =2m

P=2tonne=2000*9.81=19620 N

e = 5 cm= 0.05 m

for circular section of column

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32} \times 2^3 = 0.7853\ m^3$$

Direct compressive stress(σ_c):

We know that

$$\sigma_c = \frac{P}{A} = \frac{P}{\frac{\pi}{4} \times d^2} = \frac{19620}{\frac{\pi}{4} \times 2^2} = 6245.23 \frac{N}{m^2}$$

Bending stress induced in column (σ_b):

We know that

$$\sigma_b = \frac{M}{Z} = \frac{P \times e}{\frac{\pi}{32} \times d^3} = \frac{19620 \times 0.05}{\frac{\pi}{32} \times 2^3} = 1249.2 \frac{N}{m^2}$$

Maximum stress induced in beam due to eccentric load (σ_{max}):

$$\sigma_{max} = \sigma_c + \sigma_b = \frac{P}{A} + \frac{M}{Z} = \frac{P}{\frac{\pi}{4} \times d^2} + \frac{P \times e}{\frac{\pi}{32} \times d^3} = 6245.23 + 1249.2 \frac{N}{m^2} = 7494.43 \frac{N}{m^2}$$

$$\boldsymbol{\sigma_{max} = 7494.43 \frac{N}{m^2}}$$

5. **A rectangular column 250mm wide and 150mm. thick is carrying a vertical load of 150KN at an eccentricity of 60mm in a plane bisecting the thickness. Determine the maximum & minimum intensity of stress in the section.**

Given data:

d =250 mm

b = 150 mm

P=150 KN =150000 N

e = 60 mm

For Rectangular section of column

Area of Cross section of Column

$$A = b \times d = 250 \times 150 = 37500 mm^2$$

Section modulus of column

$$Z = \frac{db^2}{6} = \frac{250 \times 150^2}{6} = 937500 mm^3$$

Direct compressive stress(σ_c):

We know that

$$\sigma_c = \frac{P}{A} = \frac{P}{\frac{\pi}{4} \times d^2} = \frac{150 \times 10^3}{\frac{\pi}{4} \times 250^2} = 4\,\frac{N}{mm^2}$$

Bending stress induced in column (σ_b):

We know that

$$\sigma_b = \frac{M}{Z} = \frac{P \times e}{\frac{\pi}{32} \times d^3} = \frac{150000 \times 60}{\frac{\pi}{32} \times 250^3} = \frac{150000 \times 60}{\frac{\pi}{32} \times 250^3}$$

$$= 9000000/937500 = 9.6\,\frac{N}{mm^2}$$

Maximum stress induced in beam due to eccentric load (σ_{max}):

$$\sigma_{max} = \sigma_c + \sigma_b = \frac{P}{A} + \frac{M}{Z} = \frac{P}{\frac{\pi}{4} \times d^2} + \frac{P \times e}{\frac{\pi}{32} d^3} = 4 + 9.6\,\frac{N}{mm^2} = 13.6\,\frac{N}{mm^2}$$

$$\boldsymbol{\sigma_{max} = 13.6\,\frac{N}{mm^2}\,(Compressive)}$$

Minimum stress induced in beam due to eccentric load (σ_{min}):

$$\sigma_{min} = \sigma_c - \sigma_b = \frac{P}{A} - \frac{M}{Z} = \frac{P}{\frac{\pi}{4} \times d^2} - \frac{P \times e}{\frac{\pi}{32} d^3}$$

$$= 4 - 9.6\,\frac{N}{mm^2} = -5.6\,\frac{N}{mm^2}\,(Tensile)$$

$$\boldsymbol{\sigma_{min} = -5.6\,\frac{N}{mm^2}\,(Tensile)}$$

Type's column

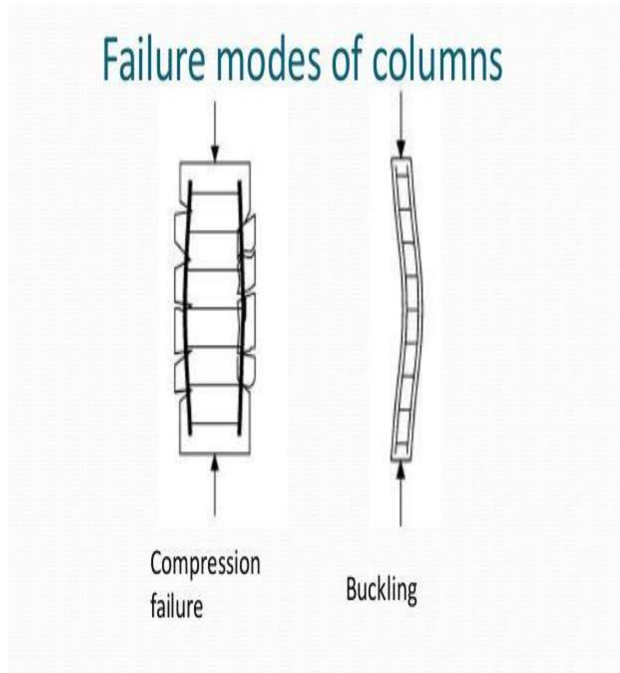
- Long Column
- Short (Strut) Column
- Medium Column

Long Column:

It is the type of column in which the length of column is more than its lateral dimensions.

In other word the ratio of Effective Length to Minimum radius of Gyration of Column is more than equal to 50

It tends to buckle while subjected to compressive load



Short Column:

It is the type of column whose length is very small as compared to the lateral dimension of Column

In other word the ratio of Effective Length to Minimum radius of Gyration of cross-section is less than equal to 50

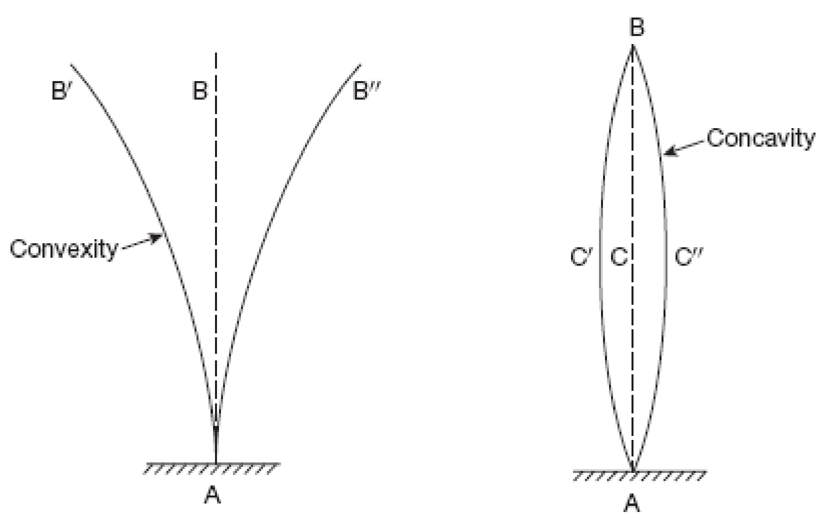
It fails by crushing due to compressive load applied on it

Medium Column:

If the ratio of Effective Length to Minimum radius of Gyration of Cross Section is more than equal to 30 and less than equal to 100

Assumptions Made in the Euler's Column Theory:

1. The column is initially perfect straight and load is applied axially
2. The cross-section of column is uniform throughout its length
3. The column material is perfectly elastic, homogeneous and Isotropic
4. The length of the column is very large compared to its lateral dimensions
5. The direct stress is very small as compared to the bending stress
6. The column will fail by buckling alone



(a) Positive
(b) Negative

7. The self-weight of column is negligible

Sign Conventions:

The following sign convention for the bending of the column will be used

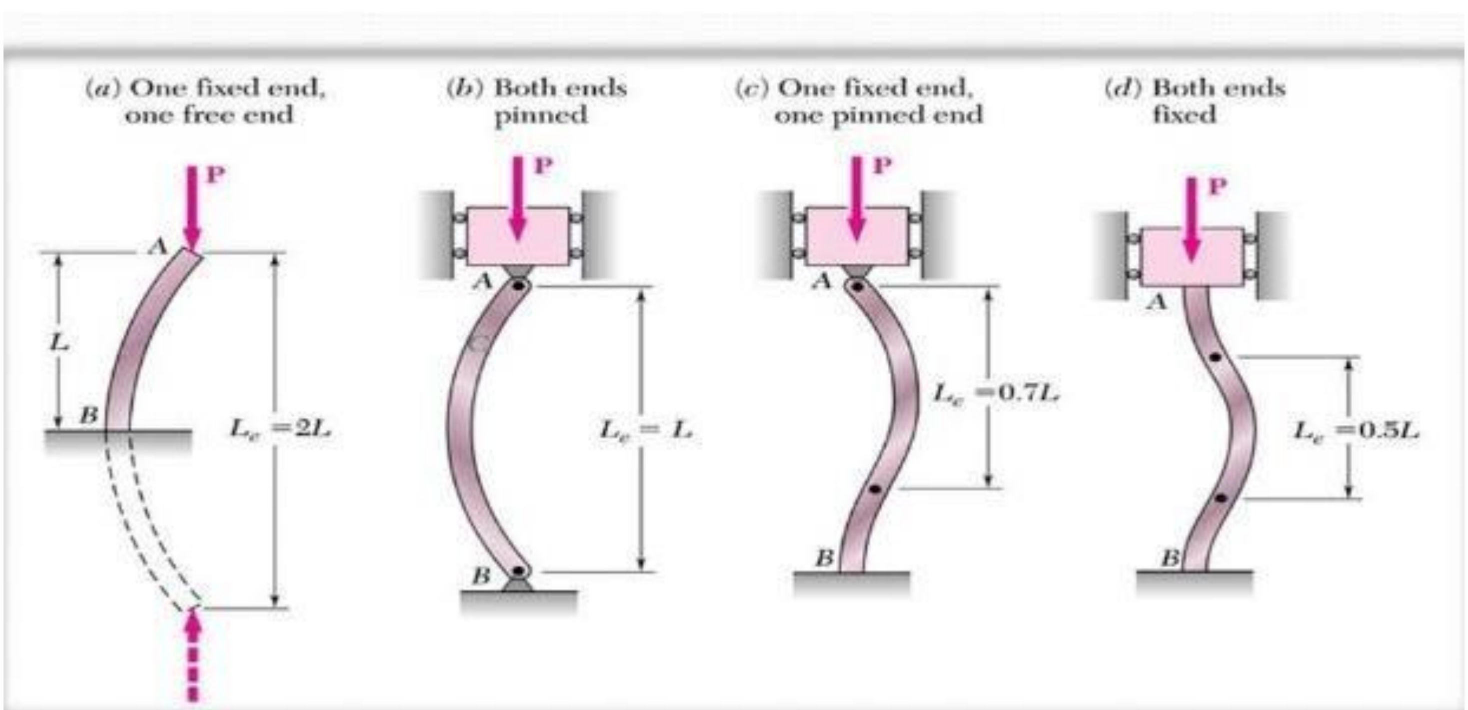
1. A moment which will bend the column with its convexity towards its initial central line as shown in fig. is taken as positive. AB represents the initial central line of column. Whether the column bends taking the shape AB'' , the moment producing this type of curvature is positive
2. A moment which will tend to bend the column with its concavity towards its initial center line as shown in figure is taken as negative

End condition of Column:

In case of Long columns the stress due to direct load is very small in comparison with the stress due to buckling. Hence the failure of long column take place entirely due to bucking for bending. The following four types of end conditions of the columns are important:

1. Two ends are fixed
2. Two ends are pinned (Hinged)
3. One end of column is fixed and other end is free
4. One end of column is fixed and other end is pinned(Hinged)

   For pinned ends there is deflection and also for fixed deflection & slope are Zero, for free end deflection is not Zero



Crippling load for various End Condition of Column:

S. NO	END CONDITION	RELATION BETWEEN EFFECTIVE AND ACTUAL LENGTH	CRIPPLING LOAD(P)
1	Both sides hinged	$L_e = L$	$P_E = (\pi^2 E I) / L_e^2$
2	One fixed and other free	$L_e = 2L$	$P_E = (\pi^2 EI) / 4L_e^2$
3	both fixed	$L_e = L/2$	$P_E = 4(\pi^2 EI) / L_e^2$
4	One fixed and other hinged	$L_e = L/\sqrt{2}$	$P_E = 2(\pi^2 EI) / L_e^2$

Chapter-7

Torsion

Shaft:

- A shaft is a rotating machine element, usually circular in cross section, which is used to transmit power from one part to another, or from a machine which produces power to a machine which absorbs power.
- The material used for ordinary shafts is mild steel. When high strength is required, an alloy steel such as nickel, nickel-chromium or chromium-vanadium steel is used.
- Shafts are generally formed by hot rolling and finished to size by cold drawing or turning and grinding

The following stresses are induced in the shafts.

1. Shear stresses due to the transmission of torque (due to torsional load).
2. Bending stresses (tensile or compressive) due to the forces acting upon the machine elements like gears and pulleys as well as the self-weight of the shaft.
3. Stresses due to combined torsional and bending loads.

Type of Shaft:

There are two types shaft

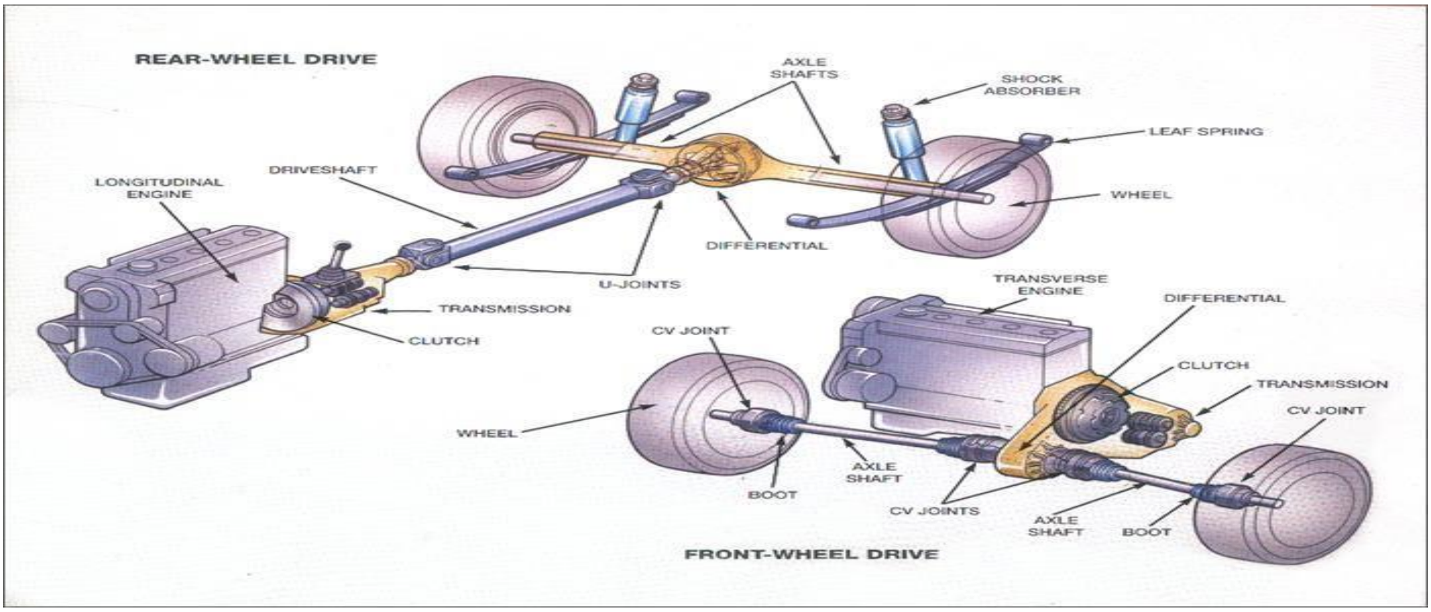
1. Transmission Shaft
2. Machine Shaft

Transmission Shaft:

These shafts transmit power between the source and machine absorbing power. The Counter shafts, line shafts, overhead shafts are called the transmission shaft

Machine Shaft:

These shaft from an integral part of the machine itself



Torsion:

A shaft is said to be in pure torsion if it is subjected to two equal and opposite couples are applied at two ends of shaft so that the shaft twist through 1 degree for its unit length.

Assumption taken in torsion:

i. The material of shaft is uniform throughout.

ii. The shaft circular in section remains circular after loading.

iii. The twist along the length of shaft is uniform through out

iv. Maximum shear stress induced in the shaft due to application of torque does not exceed its elastic limit

v. The distance between any two normal cross-section remains constant after the application o torque

Derive of Torsion Equation

Let consider a solid shaft of circular shaft is subjected to two equal and opposite twisting forces as shown in fig.one end of shaft is fixed and other end is subjected to applied couple. The twisting forces produce an angle of twist in shaft of unit length.

Let

$\mathrm{L} -$ Length of shaft in meter

$\mathrm{D} -$ Diameter of Shaft in meter

$\theta -$ angle of twist of shaft in degree

$\boldsymbol{\phi} -$ shear strain induced in shaft

$\tau -$ shear stress induced in shaft in $\frac{\mathrm{N}}{\mathrm{m}^2}$

$\mathrm{G, C} -$ modulus of Rigidity for material of shaft in ${}^{\mathrm{N}}/{}_{\mathrm{m}^2}$

$\mathrm{T} -$ Twisting moment or torque transmission by shaft

$\mathrm{J} -$ Polar moment of Inertia for circular cross section of shaft in m^4

let's consider a length of shaft 'AB' on surface of shaft before twisting force is applied on it due to application of twisting force on shaft

it is observed that the point B changes its posion to point B' andproduce a angle of twist 'θ'and shear strain in surface of shaft as '**ϕ**'

Shear strain produced in shaft

$$\tan\boldsymbol{\phi} = \frac{\text{change in length of shaft AB}}{\text{original length of shaft AB}} = \frac{\mathrm{BB'}}{\mathrm{AB}} = \frac{\mathrm{BB'}}{\mathrm{L}}$$

The change in length of shaft

$$\mathrm{BB'} = \mathrm{R}\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (\text{length of arc BB'having centre at 'o'point })$$

Now putting value of BB' in above equation

$$\tan\boldsymbol{\phi} = \frac{\mathrm{BB'}}{\mathrm{AB}} = \frac{\mathrm{BB'}}{\mathrm{L}} = \frac{\mathrm{R}\theta}{\mathrm{L}}$$

For very small angle of $\boldsymbol{\phi}$, $\tan\boldsymbol{\phi} = \boldsymbol{\phi}$

So

$$\boldsymbol{\phi} = \frac{\mathrm{R}\theta}{\mathrm{L}}$$

We know that Modulus of Rigidity of material

$$\mathrm{G} = \frac{\tau}{\boldsymbol{\phi}}$$

$$\Rightarrow \mathrm{G} = \frac{\tau}{\frac{\mathrm{R}\theta}{\mathrm{L}}}$$

$$\Rightarrow \mathrm{G} \times \frac{\mathrm{R}\theta}{\mathrm{L}} = \tau$$

$$\Rightarrow \frac{\mathbf{G}\boldsymbol{\theta}}{\mathbf{L}} = \frac{\boldsymbol{\tau}}{\mathbf{R}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{(i)}$$

From above equation it is observed that shear stress is directly proportional to Radius of shaft

$$\mathrm{i.e}\ \tau \propto \mathrm{R}$$

Twisting moment/ Torque transmission by shaft:

In order to calculate torque transmission by shaft let's consider the circular cross-section of shaft as shown by figure below. The circular section is converted into no of equal small division

To determine the twisting moment produced by couples

Consider a small ring strip of thickness 'dr' and area of cross-section 'dA' which is at a distance of 'r' from centre of shaft

We have shear stress is directly proportional to Radius of shaft

$$\text{i.e}\ \tau \propto R$$

Now shear stress acting on small strip be 'q'

Now the equivalent value of shear stress on strip

$$\frac{\tau}{R} = \frac{q}{r} \quad \Rightarrow q = \frac{\tau}{R} \times r \ \ldots\ldots\ldots\ldots\ \text{value of shear stress on shaft}$$

The twisting force acting on small strip of shaft

$$df = q \times dA$$

$$df = \frac{\tau}{R} \times r \times dA$$

$$df = \frac{\tau}{R} \times r \times (2\pi r.dr)$$

$$df = \frac{\tau}{R} \times r^2 \times 2\pi.dr$$

Torque/Turning Moment produced by force on small strip

$$dT = df \times r = \frac{\tau}{R} \times 2\pi \times r^3 \times dr$$

Now the torque transmission by whole shaft can be determined by

$$T = \int_0^R \left(\frac{\tau}{R} \times 2\pi \times r^3 \times dr\right)$$

$$T = \frac{\tau}{R} \times 2\pi \int_0^R (r^3 \times dr)$$

$$T = \frac{\tau}{R} \times 2\pi \times \left[\frac{R^4}{4} - \frac{0}{4}\right]$$

$$T = \frac{\tau}{R} \times \left(\pi \times \frac{R^4}{2}\right)$$

$$T = \frac{\tau}{R} \times \left(\pi \times \frac{\left(D/2\right)^4}{2}\right)$$

$$T = \frac{\tau}{R} \times \frac{\pi}{32} D^4 = \frac{\tau}{R} \times J$$

$$T = \frac{\tau}{R} \times J$$

$$\Rightarrow \mathbf{\frac{T}{J} = \frac{\tau}{R}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ \mathbf{(ii)}$$

Now equating both equation (i) & (ii)

$$\frac{G\theta}{L} = \frac{\tau}{R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (i)$$

$$\frac{T}{J} = \frac{\tau}{R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ (ii)$$

$$\Rightarrow \frac{\mathbf{T}}{\mathbf{J}} = \frac{\boldsymbol{\tau}}{\mathbf{R}} = \frac{\mathbf{G\theta}}{\mathbf{L}} \ \mathbf{or}\ \left(\frac{\mathbf{T}}{\mathbf{J}} = \frac{\boldsymbol{\tau}}{\mathbf{R}} = \frac{\mathbf{C\theta}}{\mathbf{L}}\right) \ldots\ldots\ldots\ldots\ldots\ldots.. \textbf{Torsion equation}$$

Where

$\mathrm{T} =$ torque transmission by shaft in Nm

$\mathrm{J} -$ Polar moment of inertia ofsection of shaft

$\tau =$ shear stress induced in shaft in $\mathrm{N}/_{\mathrm{m}^2}$

$\mathrm{R} =$ Radius of shaft in m

$\theta -$ angle of twist in degree

$\mathrm{C}, \mathrm{G} -$ Modulus of rigidity of material of shaft in $\mathrm{N}/_{\mathrm{m}^2}$

Strength of Shaft:

1. **According to shear stress induced in shaft**

The torque transmission by shaft can be obtained by

$$\frac{T}{J} = \frac{\tau}{R} \ldots\ldots \text{from torsion equation}$$

$$\Rightarrow T = \frac{\tau}{R} \times J$$

$$T = \frac{\tau}{D/2} \times \frac{\pi}{32} D^4 = \frac{\pi}{16} D^3 . \tau$$

$$\mathbf{T} = \frac{\boldsymbol{\pi}}{\mathbf{16}} \mathbf{D^3} . \boldsymbol{\tau} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{torque transmission by solid shaft}$$

For Hollow Shaft

$\mathrm{D} -$ external diameter of shaft

$\mathrm{d} -$ internal diameter of shaft

$\mathrm{J} -$ Polar moment of inertia

$\mathrm{T} -$ Torue transmission by hollow shaft

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\Rightarrow T = \frac{\tau}{R} \times J$$

$$T = \frac{\tau}{D/2} \times \frac{\pi}{32} (D^4 - d^4) = \frac{\pi}{32} \left(\frac{D^4 - d^4}{D}\right) \tau$$

$$\mathbf{T} = \frac{\boldsymbol{\pi}}{\mathbf{16}} \left(\frac{\mathbf{D^4 - d^4}}{\mathbf{D}}\right) \boldsymbol{\tau} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \textbf{torque Transmission by Hollow Shaft}$$

2. **According to Angle of Twist**

The torque transmission by shaft can be obtained by

$$\frac{T}{J} = \frac{G\theta}{L} \ldots\ldots \text{from torsion equation}$$

$$\Rightarrow T = \frac{G\theta}{L} . J$$

$$\mathbf{T} = \frac{\mathbf{G\theta}}{\mathbf{L}} \times \frac{\boldsymbol{\pi}}{\mathbf{32}} \mathbf{D^4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{torque transmission by shaft}$$

For Hollow Shaft

$\mathrm{D} -$ external diameter of shaft

$\mathrm{d} -$ internal diameter of shaft

$J -$ Polar moment of inertia

$T -$ Torue transmission by hollow shaft

The torque transmission by shaft can be obtained by

$$\frac{T}{J} = \frac{G\theta}{L} \dots \dots \text{from torsion equation}$$

$$\Rightarrow T = \frac{G\theta}{L} . J$$

$$\mathbf{T = \frac{G\theta}{L} \times \frac{\pi}{64} (D^4 - d^4) \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots . torque\ Transmission\ by\ Hollow\ Shaft}$$

Power Transmission By shaft:

The twisting moment of shaft causes the transmission of power from one place to another place by means of power transmitting devices (i.e. Gear, Belt drive & chain drive) mounted on it

Let

$N -$ speed of shaft in R. P. M

$P -$ Power Transmission by Shaft

$T -$ tourqe Transmission by shaft

we have power transmission by shaft

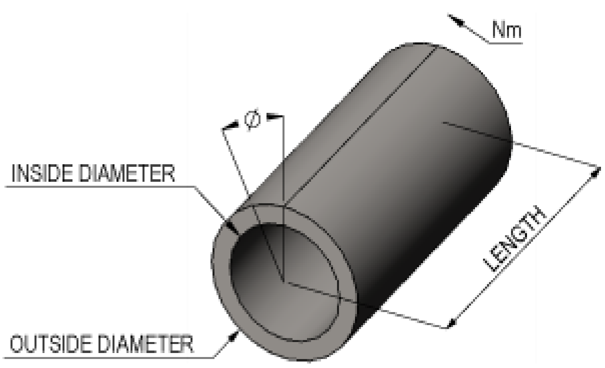
$P = T. \omega$

$$P = T\left(\frac{2\pi N}{60}\right) = \frac{2\pi NT}{60}$$

$$\mathbf{P = \frac{2\pi NT}{60 \times 1000}\ KW}$$

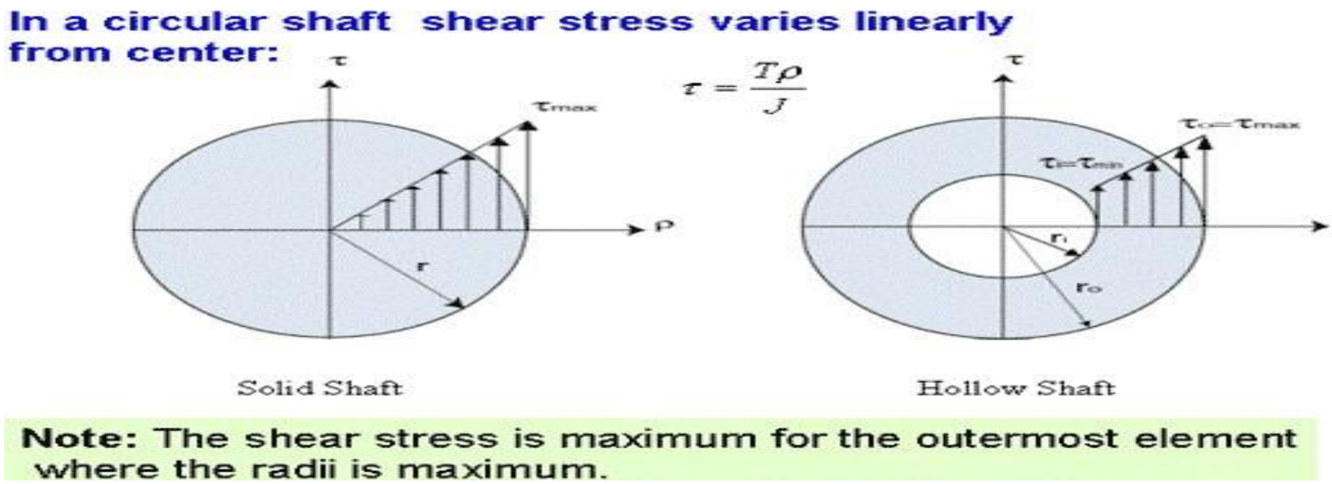
Hollow shaft vs Solid shaft:

The hollow shaft contains the same amount of material all at the ends if hollow and a solid shaft are of the same weight whereas in the solid shaft distributed the material is uniformly throughout the shaft leaves little material at the ends. Difference between both of them is resistance to bending, resistance to torsion. Let us have a deep insight into the difference between the hollow and solid shaft.



Difference between the hollow shaft and solid shaft:

- The material at the centre is removed in the hollow shaft and spread at large radius therefore, hollow shafts are stronger than solid shafts with the same weight.
- The stiffness of the hollow shaft is more than the same weight solid shaft.
- The strength of the hollow shaft is more than the same weight solid shaft.
- The natural frequency of the hollow shaft is higher than the solid shaft with the same weight.
- Hollow shaft is costlier than the solid shaft.
- The diameter of the hollow shaft is more than a solid shaft and require more space.
- Hollow shafts have a more polar moment of inertia, which enables them to transmit more torque than solid shafts.
- Hollow shafts do not transfer more power but the power to weight ratio of hollow shafts is more as compared to the solid shaft.
- Solid shafts, when subjected to bending, are stronger than that of the hollow shaft.



Torsional Rigidity:

It is defined as the amount of twisting moment required to produce unit angle of twist of 1 degree in a unit length of shaft. It is also known as torsional stiff ness.
It is defined as the product of modulus of rigidity and polar moment of inertia
Its unit Nm^2
From torsion equation

$$\frac{T}{J} = \frac{G\theta}{L} \ldots\ldots \text{from torsion equation}$$

$$\Rightarrow \frac{TL}{G.J} = \theta \ldots\ldots\ldots\ldots\ldots\ldots\text{Angle of Twist}$$

A hollow shaft is have an outside diameter'd' and inside diameter d/2. Calculate the minimum value of'd' If it is to transmit 375KW at 105rpm with a working stress 40N/mm². Determine the twist in a length equal to 10times the external dia. Take C=8x10⁴ N/mm²

Given Data:

$\text{Power Transmission by shaft } P = 375 \text{ KW} = 375 \times 10^6 \, {}^{\text{Nmm}}/_{\text{Sec}}$

$\text{Outer Diameter } D_o = d$

$\text{Internal diameter } D_i = \frac{d}{2}$

$\text{Speed of shaft } N = 105 \text{ rpm}$

$\text{Shear stress } \tau = 40 \, {}^{\text{N}}/_{\text{mm}^2} \text{ Length of Shaft } L = 10 \, D_o$

$\text{Angle of twist } \theta = ?$

We have Power transmission by shaft

$$P = \frac{2\pi NT}{60} = 375 \times 10^6$$

$$\Rightarrow \frac{375 \times 10^6}{2\pi N} = T$$

$$\Rightarrow T = \frac{375 \times 10^6}{2\pi \times 105} = 568410.51 \text{ Nmm}$$

We have for a hollow shaft Torque Transmission by shaft

$$\mathbf{T} = \frac{\pi}{32}\left(\frac{D_o^4 - D_i^4}{D_o}\right)\tau = 568410.51$$

$$\Rightarrow \left(\frac{D_o^4 - D_i^4}{D_o}\right) = \frac{568410.51 \times 32}{\pi} = 5789781.911$$

$$\Rightarrow \left(\frac{d^4 - {d/_2}^4}{d}\right) = 5789781.911$$

$$\Rightarrow \left(\frac{16 \times d^4 - d^4}{16 \times d}\right) = \frac{15 \times d^3}{16} = 5789781.911$$

$$\Rightarrow d = \sqrt[3]{\frac{5789781.911 \times 16}{15}} = 42.57 \text{ mm}$$

$$\Rightarrow \mathbf{d = 42.57\ mm} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \textbf{External Diameter of Hollow Shaft } (\mathbf{D_o})$$

$\textbf{internal Diameter of Shaft } \mathbf{D_i} = {\mathbf{d}/_\mathbf{2}} = {\mathbf{42.57}/_\mathbf{2}} = \mathbf{21.285\ mm}$

$\textbf{length of shaft } \mathbf{L = 10 \times d = 10 \times 42.57 = 425.7\ mm}$

Polar moment inertia of Hollow Shaft

$$J = \frac{\pi}{32}(D_o{}^4 - d_i^4)$$

$$J = \frac{\pi}{32}(42.57^4 - 21.285^4) = 302263.54 \text{ mm}^4$$

We have From Torsion Equation

$$\frac{T}{J} = \frac{G\theta}{L} \ldots\ldots \text{from torsion equation}$$

$$\Rightarrow \frac{TL}{JG} = \theta$$

$$\Rightarrow \frac{568410.51 \times 425.7}{302263.54 \times 8 \times 10^4} = \theta$$

$$\Rightarrow \boldsymbol{\theta} = \mathbf{0.010006\ Rad\ say\ 0.5733^\circ} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{Angle of Twist of shaft}$$

A Hollow shaft of diameter ratio 3/8 is required to transmit 600KW at 110rpm. The maximum shear torque 20% greater than the mean. The shear stress is not to exceed 63 MN/m^2 and the twist in a length of 3m' not to exceed 1.4 degrees calculate maximum external dia. Satisfying these conditions. C=84GN/m^2

[2013w, 2012w, 2011w, 2010w]

Given Data:

Power Transmission by shaft $P = 600\ KW = 600\ \times 10^6\ ^{Nmm}/_{Sec}$

Outer Diameter $D_o = 0.375D_i$

Speed of shaft $N = 110$ rpm

Maximum Torque $T_{Max} = 1.2\ T_{Mean}$

Shear stress $\tau = 63\ ^{N}/_{mm^2}$

Length of Shaft $L = 3m = 3000$ mm angle of twist $\theta = 1.4° = 0.0244$ rad

Angle of twist $\theta = ?$

We have Power transmission by shaft

$$P = \frac{2\pi NT}{60} = 600 \times 10^6$$

$$\Rightarrow \frac{600 \times 10^6}{2\pi N} = T$$

$$\Rightarrow T = \frac{600 \times 10^6}{2\pi \times 110} = 868117.87\ \text{Nmm}$$

We have for a hollow shaft Torque Transmission by shaft

$$\mathbf{T} = \frac{\pi}{32}\left(\frac{D_o^4 - D_i^4}{D_o}\right)\tau = 868117.87$$

$$\Rightarrow \left(\frac{D_o^4 - D_i^4}{D_o}\right) = \frac{868117.87 \times 32}{\pi} = 8842576.026$$

$$\Rightarrow \left(\frac{D_o^4 - 0.375D_o^4}{D_o}\right) = 8842576.0626$$

$$\Rightarrow (D_o^3) = \frac{8842576.0626}{0.9802}$$

$$\Rightarrow D_o = \sqrt[3]{9020969.216} = 208.16\ \text{mm}$$

$$\Rightarrow \mathbf{D_i = 78.06\ mm} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . \textbf{Internal Diameter of Hollow Shaft } (\mathbf{D_o})$$

External Diameter of Shaft $D_0 = 208.16$ mm

Polar moment inertia of Hollow Shaft

$$J = \frac{\pi}{32}(D_o^4 - D_i^4)$$

We have From Torsion Equation

$$\frac{T}{J} = \frac{G\theta}{L} \dots\dots \text{from torsion equation}$$

$$\Rightarrow \frac{TL}{G\theta} = J$$

$$\Rightarrow \frac{868117.87 \times 3000}{0.0244 \times 8 \times 10^4} = J$$

$$\Rightarrow \mathbf{J = 1334196.20}$$

$$J = \frac{\pi}{32}(D_o{}^4 - D_i{}^4) = 1334196.20$$

$\Rightarrow (D_o{}^4 - D_i{}^4) = {}^{1334196.20 \times 32}/_{\pi} = 13590010.89$

$(D_o{}^4 - (0.375 D_o{}^4)) = {}^{1334196.20 \times 32}/_{\pi} = 13590010.89$

$D_o = \sqrt[4]{13864180.48} = 61.02 \text{ mm}$

$\mathbf{D_o = 61.02\ mm} \dots\dots\dots\dots$ **External Diameter of shaft**

Internal Diameter of Shaft $\mathbf{D_i = 22.88\ mm}$